

# A New Design for WDM Packet Switching Networks with Wavelength Conversion and Recirculating Buffering

Zhengkao Zhang and Yuanyuan Yang

Department of Electrical & Computer Engineering

State University of New York, Stony Brook, NY 11794, USA

**Abstract**—In this paper we study switching fabric design in WDM optical switching networks with recirculating buffers. The switching network we consider may have arbitrary wavelength conversion capabilities. We focus on limited range wavelength conversion while considering full range wavelength conversion as a special case. We show that by adopting the idea of concentrators the cost of the switching fabric can be substantially reduced. For example, for a typical switching network with 16 input/output fibers, 16 wavelength channels per fiber, 12 delay lines and wavelength conversion degree 7, the new design can yield about 20% savings in network cost. We also give an efficient algorithm to assign the outputs to the inputs in the switching network.

## I. INTRODUCTION AND BACKGROUND

All optical networking has been proposed as a promising candidate for high-speed communication networks [7], [3], [5] because of the huge bandwidth of optics: a single fiber has a bandwidth of nearly 50 THz [10]. To fully utilize the bandwidth, the bandwidth of a fiber can be divided into a number of independent channels, with each channel on a different wavelength. This is referred to as *wavelength-division-multiplexing(WDM)*.

In this paper, we study time slotted WDM packet switching networks as it may offer better flexibility and better exploitations of the bandwidth [7]. In particular, we study switching fabric design of a WDM switch or a WDM interconnect with recirculating buffer pool and limited range wavelength conversion. As in [7] [3] [5], we assume that the duration of an optical packet is one time slot and the traffic pattern is unicast, i.e., each packet is destined to only one output fiber.

In a WDM switch, output contention occurs when more than one packets on the same wavelength are destined to the same output fiber. To resolve the contention, we can send the contending packets to optical buffers made of fiber delay lines to delay them for a certain amount of time. Fiber delay lines are costly and bulky. Therefore, instead of using dedicated buffers for each output fiber as in [3] [5], we consider WDM switch with shared buffers which require fewer number of buffers due to statistical multiplexing. In such a switch, packet that cannot be directly sent to the output fiber

is first sent to a delay line. After being delayed for some time, the packet will come out of the delay line and attempt to be transmitted again. If fails one more time, the packet will again be sent to a delay line to wait for the next round.

Another method to resolve output contention is to convert the wavelength of a packet to some other idle wavelength. We consider *limited range* wavelength converter which is capable of converting a wavelength to a limited number of wavelengths, since it is more realistic and cost-effective than *full range* wavelength converter which is capable of converting a wavelength to any wavelength [8], [4], [9]. Also, full range wavelength converters can be regarded as a special case of limited range wavelength converters.

Wavelength converters can be “variable input fixed output” which converts several wavelengths to one fixed wavelength, or “variable input variable output” which converts several wavelengths to several other wavelengths [6]. The probe signal is a laser on a fixed wavelength for the former and is a tunable laser for the latter. Therefore, the first type of wavelength converters are much cheaper than the second type. In a WDM switching network, if the first type of wavelength converter is used, it will be placed right in the front of each output wavelength channel. All the input wavelength channels that can be converted to this output wavelength will first be connected to this converter. At one time slot only one of these inputs is active, and its wavelength will be converted to this output wavelength by the converter. If the second type of wavelength converter is used, it will be placed directly following each input wavelength channel. The input wavelength is first converted to a proper wavelength by the converter and then sent to the output fiber. The output of the wavelength converter is only connected to each output fiber but not to all the convertible wavelength channels. Thus the cost of the switching fabric for the second type will be smaller than the first type. However, if the conversion degree is small, i.e., the number of convertible wavelength channels is small, the overall cost of the entire switch of the first type may still be less than that of the second type. In this paper we will consider the first type of wavelength converters and will give efficient switch designs to reduce the cost of the switching fabric.

Fig. 1 shows such a switching network. It has  $N$  input fibers and  $N$  output fibers. Inside the network there are also  $B$  delay lines, each capable of delaying a packet for one time slot. On each fibers there are  $k$  wavelengths. It can be seen that there are totally  $(N + B)k$  inputs to the internal switch-

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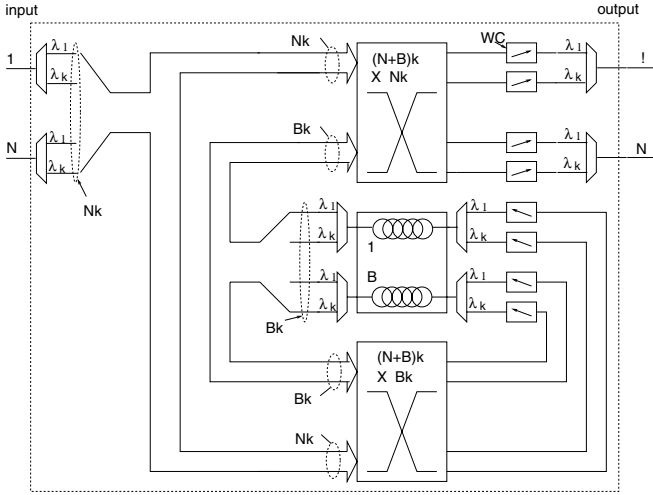


Fig. 1. Optical packet switch with shared recirculating buffers.

ing fabric,  $Nk$  from the input fibers and  $Bk$  from the recirculating buffers. These inputs can be connected to  $(N+B)k$  channels,  $Nk$  on the output fibers and  $Bk$  on the delay lines. One way of implementing the switching fabric (not shown here) is to build an  $(N+B)k \times (N+B)k$  switching network. However, this may not be necessary since a packet that should be buffered does not need to specify which delay line it should be directed to and it can be any one of them. As a result, the switching fabric connecting the inputs to the delay lines needs not be so “powerful” as the switching fabric connecting the inputs to the output fibers. Hence, as illustrated in Figure 1, we can build two switching fabrics, the upper one is  $(N+B)k \times Nk$  which connects the inputs to the output fibers and the lower one is  $(N+B)k \times Bk$  which connects the inputs to the delay lines. We will show that we can adopt the idea of *concentrators* to reduce the cost of the lower switching fabric.

## II. WAVELENGTH CONVERSION

As mentioned earlier, with limited range wavelength conversion, an incoming wavelength may be converted to a set of adjacent outgoing wavelengths. We define the set of these outgoing wavelengths as the *adjacency set* of this input wavelength. The cardinality of the adjacency set is the *conversion degree* of this wavelength. We also define the *conversion distance* as the largest difference between the index of a wavelength and the index of a wavelength that it can be converted to.

The wavelength conversion considered in this paper can be called “ordered interval” because the adjacency set of any wavelength can be represented by an interval of integers, and intervals for different wavelengths are “ordered”. To be specific, we have the following assumptions:

*Assumption 1:* The wavelengths in the adjacency set of  $\lambda_i$ ,  $i \in [1, k]$  can be represented by an interval of integers denoted by  $[begin(i), end(i)]$ , where  $begin(i)$  and  $end(i)$  are positive

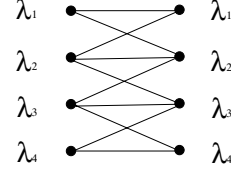


Fig. 2. Wavelength conversion of a 4-wavelength system.

integers.

*Assumption 2:* For two wavelengths  $\lambda_i$  and  $\lambda_j$ , if  $i < j$ , then  $begin(i) \leq begin(j)$  and  $end(i) \leq end(j)$ .

Figure 2 shows the conversion of a 4-wavelength system. The conversion distance for all wavelengths is 1, thus we call it “regular” wavelength conversion. We mainly consider regular wavelength conversion in this paper. However, it is not difficult to generalize our results to “irregular” wavelength conversion systems where different wavelengths may have different conversion distances. Note that full range wavelength conversion can simply be considered as a special case by letting the conversion degrees for all wavelengths be  $k$ .

## III. EFFICIENT DESIGN OF THE SWITCHING FABRIC

### A. Switching Fabric 1

The switching fabric we will design has  $(N+B)k$  inputs and  $Bk$  outputs, as there are  $N+B$  input fibers and  $B$  output fibers. Therefore for each wavelength, there are  $N+B$  inputs and  $B$  outputs. We can order the inputs and outputs according to the wavelengths, namely, the  $[t(N+B)+p]_{th}$  input is wavelength channel  $\lambda_t$  on the  $p_{th}$  input fiber and the  $[tB+p]_{th}$  output is wavelength channel  $\lambda_t$  on the  $p_{th}$  output fiber. We use  $a_i$  where  $1 \leq i \leq (N+B)k$  and  $b_u$  where  $1 \leq u \leq Bk$  to denote the inputs and outputs. With wavelength conversion ability, an input wavelength channel can be connected to the output wavelength channels it is allowed to convert to. An example of the switching fabric is shown in Figure 3, where  $N=4$ ,  $B=3$ ,  $k=4$  and the wavelength conversion is as defined in Figure 2. Each vertical line represents an input and each horizontal line represents an output. There is a node (either in black or in cyan) at the intersection of input line  $a_i$  and output line  $b_u$  if there can be a connection between  $a_i$  and  $b_u$ , i.e., the wavelength of  $b_u$  can be converted from the the wavelength of  $a_i$ , otherwise the intersection is left blank. Apparently, if the wavelength conversion is full range there will be a crosspoint at every intersection.

A possible design of the switching fabric is to let each node in Figure 3 be a crosspoint. We will call this design “Switching Fabric 1.” However, as will be seen, for our purpose, not all the crosspoints in Switching Fabric 1 are needed. Before we present a better design, we need to consider how to assign outputs for a given set of inputs in a switching fabric.

### B. First Available Algorithm

In this subsection we give an efficient algorithm to assign outputs for a given set of inputs. First we define the *feasible*

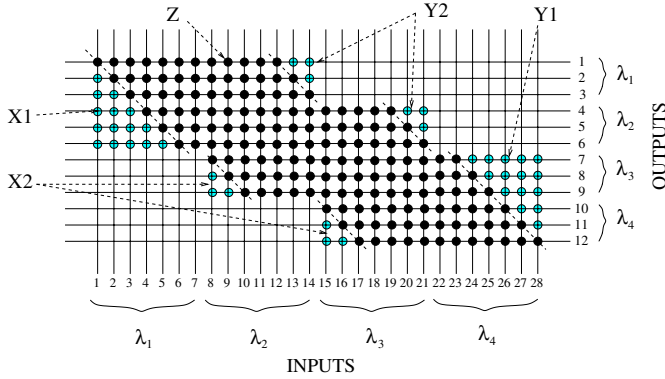


Fig. 3. Crosspoint layout for a switching fabric,  $N = 4$ ,  $B = 3$ ,  $k = 4$  and the wavelength conversion as defined in Figure 2.

TABLE 1  
FIRST AVAILABLE ALGORITHM

<p><b>First Available Algorithm</b>  <b>for</b> <math>i := 1</math> <b>to</b> <math>n</math> <b>do</b>          let <math>b_j</math> be the output adjacent to <math>a_i</math> with the          smallest index and is not assigned to any input          <b>if</b> no such <math>b_j</math> exists              <math>a_i</math> is not assigned to any output          <b>else</b>              assign <math>b_j</math> to <math>a_i</math>          <b>end if</b>  <b>end for</b></p>
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input.

*Definition 1:* A group of inputs is *feasible* to a switching fabric if each of the inputs can be assigned to some output by the switching fabric.

For Switching Fabric 1, if the wavelength conversion is full range, a group of inputs is feasible if and only if the total number of inputs is no more than  $Bk$ . When the wavelength conversion is limited range, the necessary and sufficient condition is more complicated. It will be a set of inequalities that can be derived by Hall's theorem. Luckily we do not need to use these inequalities in our designs.

We call the set of outputs that can be connected to an input the *adjacency set* of this input. Using similar arguments in [11], it can be shown that for any switching fabric, as long as the adjacency set of inputs has the following two properties, the First Available Algorithm described in Table 1 can be used to find the maximum number of inputs that can be connected simultaneously.

*Property 1:* The adjacency set of any input is an interval: We can use interval  $[begin(a_i), end(a_i)]$  to represent the adjacency set of left side vertex  $a_i$ .

*Property 2:* If  $i < j$  then  $begin(a_i) \leq begin(a_j)$  and  $end(a_i) \leq end(a_j)$ .

The time complexity of the First Available Algorithm is

$O(n)$  where  $n$  is the number of inputs. The algorithm can be used on Switching Fabric 1 since the adjacency set of the inputs has these two properties due to the assumptions on wavelength conversion. If the set of inputs is feasible, the algorithm assigns an output for each of the inputs.

### C. Switching Fabric 2 – Concentrator

We now adopt the idea of concentrators to further reduce the switch cost. A  $p \times q$  concentrator is a switching fabric with  $p$  inputs and  $q$  outputs where  $p > q$ , and for any  $q$  inputs, it is capable of assigning an output for each of the inputs [13], [14]. However, it is not guaranteed that an input can be connected to a specific output. The concentrator only guarantees that there exists some output that this input can be connected to.

The switching fabric we are designing can also be a concentrator, since an input packet needs only to be assigned to some wavelength channel and this wavelength channel can be on any delay lines. However, because of the wavelength conversion constraint, the “concentrator” here is more complicated than an electronic concentrator. We define a WDM concentrator with limited range wavelength conversion as follows.

*Definition 2:* A WDM switching fabric with limited range wavelength conversion is a concentrator if any feasible input to Switching Fabric 1 is also feasible to it.

We now divide the crosspoints of Switching Fabric 1 into three areas, *area X*, *area Y* and *area Z*, as shown in Figure 3, where area X contains the triangle areas with cyan nodes in the lower half of the switching fabric, area Y contains the triangle areas with cyan nodes in the upper half symmetrical to area X, and area Z contains the rest of the switching fabric. Next we will show that a concentrator needs only the crosspoints in area Z.

We now give formal definitions for these areas. There are two sub areas of area X, X1 and X2. X1 is a single larger triangle area and X2 is a group of smaller triangle areas. Imagine drawing a 45 degree line crossing nodes starting from  $(a_1, b_1)$ , and extending to  $(a_2, b_2)$ ,  $(a_3, b_3)$ , ..., till  $(a_i, b_i)$  (In Figure 3,  $i = 6$ , or the line stops at node  $(a_6, b_6)$ ). The line will stop at  $(a_i, b_i)$  because node  $(a_{i+1}, b_{i+1})$  does not exist. Nodes under this line are said to be in area X1. If node  $(a_{i+1}, b_{i+1})$  does not exist but  $(a_i, b_i)$  does, either  $b_{i+1}$  does not exist or  $b_i$  is the end of the adjacency interval of  $a_{i+1}$ . If it is the first case,  $i = kB$  and there is no area X2. Otherwise let  $i = tB$  for some positive integer  $t$ . In Figure 3 area X2 exists and  $t = 2$ . In general,  $t$  satisfies

$$\left\lceil \frac{Bt}{N+B} \right\rceil + d = t \quad (1)$$

where  $d$  is the conversion distance. For the  $B$  outputs on  $\lambda_{t+1}$ :  $b_{tB+1}, b_{tB+2}, \dots, b_{tB+B}$ , let  $a_{w+1}, a_{w+2}, \dots, a_{w+B}$  be the first (with the smallest index)  $B$  inputs adjacent to them where  $w = h(N+B)$  for some positive integer  $h$ . In the example  $h = 1$ . The two parameters,  $t$  and  $h$ , will be used lat-

er in our proofs. Imagine drawing a 45 degree line crossing  $n$ -nodes  $(a_{w+1}, b_{tB+1}), (a_{w+2}, b_{tB+2}), \dots, (a_{w+B}, b_{tB+B})$ .  $n$ -nodes under this line are said to be in area X2 for output wavelength  $\lambda_{t+1}$ . In Figure 3, this line crosses  $(a_8, b_7), (a_9, b_8), (a_{10}, b_9)$  and nodes in area X2 for output wavelength  $\lambda_3$  are  $(a_8, b_8), (a_8, b_9), (a_9, b_9)$ . Area X2 for output wavelengths  $\lambda_{t+2}, \lambda_{t+3}, \dots, \lambda_k$  can be similarly defined. The definitions for areas Y1 and Y2 are similar to areas X1 and X2, except that the imaginary 45 degree line starts from the other end (node  $(a_{(N+B)k}, b_{Bk})$ ) and the nodes *above* the line are in area Y.

If the First Available Algorithm is run on Switching Fabric 1, we have:

*Lemma 1:* The First Available Algorithm does not use the crosspoints in area X.

**Proof.** Consider area X1 first. If  $a_i$  is assigned to  $b_u$  by the First Available Algorithm, all  $b_1$  to  $b_{u-1}$  are not available to  $a_i$  when  $a_i$  is checked. This only occurs when all these outputs are assigned to some inputs with smaller indexes than  $a_i$  by the algorithm. But this cannot happen since there are at most  $i-1$  inputs before  $a_i$  and  $u > i$ .

Now consider the first block of area X2, the area for  $\lambda_{t+1}$ . Suppose node  $(a_j, b_v)$  is in area X2 and  $a_j$  was assigned to  $b_v$  by the algorithm. Note that inputs from  $a_1$  to  $a_{h(N+B)}$  can only be assigned to  $b_1$  to  $b_{tB}$ . Thus if  $a_j$  is assigned to  $b_v$ ,  $b_{tB+1}$  to  $b_{v-1}$  must be all assigned to inputs from  $a_{h(N+B)+1}$  to  $a_{j-1}$ . This cannot happen since  $j - h(N+B) < v - tB$  if  $(a_j, b_v)$  is in area X2. Similarly we can prove for other blocks in area X2. ■

*Lemma 2:* After running the First Available Algorithm on the Switching Fabric 1, any assignments in area Y can be swapped into the area Z without any collision.

**Proof.** Consider area Y1. If there are assignments in this area, let  $a_i$  be the one with the largest index and suppose it is assigned to  $b_u$ . By the algorithm, outputs from  $b_{u+1}$  to  $b_{kB}$  are not assigned to any inputs. Hence we can assign  $a_i$  to  $b_B$ . Thus  $b_u$  to  $b_{kB-1}$  will be free and we can assign the active input with the second largest index,  $a_j$ , to  $b_{kB-1}$ , and  $b_v$  to  $b_{B-2}$  will be free, where  $b_v$  was assigned to  $a_j$ . This swapping can be carried on, and in the  $p$ th swapping the active input with the  $p$ th largest index will be assigned to  $b_{kB-p+1}$ . Note that all new assignments are in area Z. There will be at most  $tB-1$  inputs that need to be swapped, and they can be swapped to  $b_{B(k-t)+2}$  to  $b_{kB}$ . Therefore the swapping can always be done. Also note that after the swapping the input will be assigned to an output with a larger index than the output it was assigned. Thus they will not collide with the assignments for inputs with smaller indexes (inputs from  $a_1$  to  $a_{k(N+B)-tB+1}$ ).

Now consider the first block of area Y2. The outputs of this block are on wavelength  $\lambda_{k-t}$ . Following similar arguments we can show that the assignments in this Y2 block can be swapped into nodes in area Z, or more specifically, nodes on the 45 degree line and on the “lower half” of the 45 degree line. The new assignments are all on wavelength  $\lambda_{k-t}$ .

They will not collide with the assignments for inputs with larger indexes than  $(k-h)(N+B)$  since these inputs are not adjacent to outputs on wavelength  $\lambda_{k-t}$  or smaller (recall that  $h$  is the parameter used in the definition of the X/Y areas and in Figure 3  $h=1$ ). For a similar reason mentioned in the proof for area Y1, the new assignments will also not collide with the assignments for inputs with smaller indexes than  $(k-h)(N+B) - B + 1$ . The proof for other Y2 areas is similar. ■

We call the switching fabric with crosspoints only in area Z “Switching Fabric 2.” We have

*Theorem 1:* Any feasible inputs to Switching Fabric 1 is also feasible to Switching Fabric 2. As a result, Switching Fabric 2 is a concentrator.

**Proof.** For any feasible input to Switching Fabric 1, an assignment can be found using the First Available Algorithm. By Lemma 1, this assignment does not contain any crosspoints in area X. And by Lemma 2, any crosspoints in area Y can be swapped into area Z. Therefore, it is also feasible to Switching Fabric 2. ■

An efficient algorithm is needed to assign outputs to inputs in Switching Fabric 2. Luckily we can simply use the First Available Algorithm, since the adjacency set of inputs in Switching Fabric 2 also has Properties 1 and 2. Moreover, the adjacency set of outputs in this switching fabric also has Properties 1 and 2. Therefore, we can run the First Available Algorithm “on the output side” and the time complexity is  $O(Bk)$ .

Theorem 2 indicates that Switching Fabric 2 uses the minimum number of crosspoints for this type of crosspoint layout.

*Theorem 2:* Switching Fabric 2 will not be a concentrator if any crosspoint is removed from it.

**Proof.** We prove this by showing that if any one of the crosspoints is removed, the remaining switching fabric will fail to realize a set of feasible inputs. For notational convenience, we define “area X inputs”. Input  $a_j$  is an area X input if the 45 degree lines used in the definition for area X crosses some node  $(a_j, b_v)$ . In the example of Figure 3 they are inputs 1-6, 8-10, and 15-17. Similarly define “area Y inputs” as the inputs involved in the 45 degree lines for the definition of area Y. In the example they are inputs 12-14, 19-21, and 23-28.

Now suppose crosspoint  $(a_i, b_u)$  was removed and that  $b_u$  is on wavelength  $\lambda_s$ . Consider the inputs which consist of all the area X inputs for output wavelengths with smaller indexes than  $s$ , all the area Y inputs for output wavelength with larger indexes than  $s$ , all the area X inputs on output wavelength  $\lambda_s$  which are not adjacent to  $b_u$ , and all the area Y inputs on output wavelength  $\lambda_s$  which are not adjacent to  $b_u$ , plus input  $a_i$ . For example, in Figure 3, if crosspoint  $(a_6, b_5)$  was removed, the set of inputs will be  $a_1$  to  $a_4$ ,  $a_{21}$ ,  $a_{23}$  to  $a_{28}$ , plus  $a_6$ . It is not difficult to verify that the outputs except  $b_u$  must be assigned to the inputs except  $a_i$ . As a result, no output can be found for  $a_i$ . But if  $(a_i, b_u)$  was not removed  $b_u$  can be assigned to  $a_i$ . ■

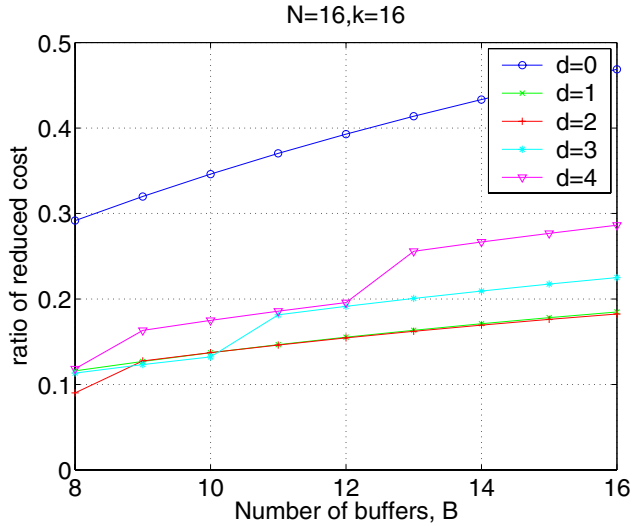


Fig. 4. Cost saved by using the concentrator design.

#### D. Cost Comparison

In this subsection, we will see how much this concentrator design reduces the switching fabric cost in terms of the number of crosspoints. Note that the number of nodes in area X as well as in area Y is

$$\alpha = kB(B-1)/2 + B^2t(t-1)/2 \quad (2)$$

where  $t$  is given in Equation (1). The number of crosspoints in Switching Fabric 1 is

$$\beta = B(N+B)[k + 2kd - d^2 - d] \quad (3)$$

assuming  $k > 2d$ . The saved cost is  $2\alpha/\beta$ . Figure 4 shows the saved cost by using Switching Fabric 2 when  $N = 16$ ,  $k = 16$  for different  $B$  and  $d$ . We can see that in general the concentrator design will save more for a larger  $B$  and a smaller  $d$ . We can also see that at least about 10% of the cost can be reduced. For example, when  $d = 3$  and  $B = 12$ , about 20% of the cost is reduced.

#### IV. CONCLUSIONS

In this paper we have proposed a new switching fabric design for WDM optical switching networks with limited range wavelength conversion. We have considered recirculating buffers and made use of the fact that the packets can be directed to any one of the delay lines in the buffer to reduce the cost of the switching fabric. The proposed design is capable of finding an output for any input in a set of feasible inputs. The First Available Algorithm can be used for scheduling in such switching fabric which runs in  $O(Bk)$ , where  $B$  is the number of delay lines and  $k$  is the number of wavelengths per fiber. We also gave formulas for calculating the percentage of the saved cost. Our future work includes the performance evaluation of the newly proposed switching fabric under different traffic.

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