Robust Design for the IRS-Assisted Multicast Communications with Statistical CSI Errors

Bolin Chen*, Jiangtian Nie†, Weiheng Jiang*, Xiaonan Zhang‡and Zehui Xiong§
*School of Microelectronics and Communication Engineering, Chongqing University, Chongqing, China
Email: chenbolin@cqu.edu.cn; whjiang@cqu.edu.cn
†School of Computer Science and Engineering, Nanyang Technological University, Singapore
Email:jnie001@e.ntu.edu.sg
‡Department of Computer Science, Florida State University, Florida, USA
Email:xzhang@cs.fsu.edu
§Pillar of Information Systems Technology and Design, Singapore University of Technology and Design, Singapore
Email:zehui_xiong@sutd.edu.sg

GLOBECOM 2022 - 2022 IEEE Global Communications Conference | 978-1-6654-3540-6/22/$31.00 ©2022 IEEE | DOI: 10.1109/GLOBECOM48099.2022.10001058

***Abstract*—Intelligent reflecting surface (IRS) is considered as an effective technology to enhance the performance of wireless communication systems. In this paper, the robust optimization design of the IRS-assisted wireless multi-group multicast MISO system with statistical CSI errors is investigated. Two optimization problems, namely max-min fairness problem and QoS problem, are discussed separately. In order to deal with the non-convex imperfect CSI constraint, the Bernstein-type inequality is utilized to transform the outage probability constraint into a second-order cone (SOC) constraint and linear inequalities. Furthermore, two efficient algorithms based on alternating optimization (AO) are proposed to solve the reformulated problems, respectively. In particular, the semi-definite relaxation (SDR) technique is applied to optimize the transmit beamforming and IRS reflection coefficients. The numerical simulation results indicate that by deploying IRS and utilizing the proposed algorithms, the system performance can be improved significantly. However, the gain of introducing IRS in the system heavily depends on the bound of the CSI error.**

***Keywords*—Intelligent reflecting surface, multi-group multicast, fairness, QoS, beamforming, imperfect CSI.**

I. Introduction

In recent years, a reconfigurable planar array technology called intelligent reflecting surface (IRS) has attracted extensive attentions from scholars. The IRS can intelligently control both the phase shift and amplitude of reflected electromagnetic waves incident on it in a programmable manner. With the help of IRS, wireless communication spectrum and energy efficiency can be improved without significantly increasing system deployment costs [1]. In view of the special functional properties of IRS, the IRS-assisted wireless communication system often confronts the non-ideal issues, like imperfect channel state information (CSI). Therefore, in order to promote the practical deployment and application of IRS, the robust optimization design under non-ideal conditions is attracting much attentions.

Preliminary research has been done on the performance analysis and optimization of IRS-enabled wireless communication systems with imperfect CSI. [2] considered an IRS-assisted MIMO system, where the beamforming at the base station (BS) and phase shifts at the IRS were optimized to maximize the average sum rate of the system, and a penalty dual decomposition (PDD) based robust design was proposed. [3] focused on the impact of CSI errors on the outage performance of multi-user downlink communication systems, where the transmit precoding vector at the access point (AP) and the discrete phase shift at the IRS were jointly optimized to minimize the transmit power. [4] studied the robust beamforming design of IRS-assisted cognitive radio (CR) system with cascade CSI errors, which jointly optimized the transmit precoding matrix and IRS phase shifts. [5] investigated the worst-case robust beamforming design in an IRS-aided multi-user MISO system, and [6] discussed the resource allocation problem of multiple IRSs-assisted multi-user MISO systems.

Furthermore, for the problem of IRS-assisted wireless multi-group multicast communications, only a few researches has been carried out at present. Specifically, [7] considered an IRS-assisted downlink multicast system, and maximized the sum rate of all multicast groups by jointly optimizing the precoding matrix of the BS and the reflection coefficient of the IRS. [8] discussed a MISO secure communication problem, where the transmit power was minimized under the secrecy rate constraint. [9] studied a multicast SISO system and proposed a novel stochastic passive beamforming scheme in which the IRS performed $Q \geq 1$ independent random reflections in each channel coherence interval without acquiring CSI.

Although there have been researches on IRS-assisted wireless communication systems under imperfect CSI, the studies on the IRS-assisted wireless multi-group multicast problem are still based on perfect CSI conditions. The more practical problem of IRS-assisted robust multicast system has not been discussed yet. The purpose of this paper is to study the wireless multi-group multicast robust system, and discuss the fairness problem and QoS problem under the statistical CSI error model. The Bernstein-type inequality is adopted to transform the outage probability constraint into a second-order cone (SOC) constraint and linear inequalities. Then, an alternating optimization (AO) based algorithm is proposed for the reformulated problems. Finally, numerical simulations show that the introduction of IRS and the proposed algorithm can significantly improve the system performance.

II. System model and imperfect CSI models

Consider the IRS-assisted downlink wireless multi-group multicast system, as shown in Fig. 1. The BS is equipped with N antennas, and the IRS adopts a uniform planar array structure with M reflection elements. Besides, there are K users in the system each with a single antenna. Define the multi-group set as $\mathcal{G} = \{\mathcal{G}_1, \mathcal{G}_2, ..., \mathcal{G}_G\}$, where $\mathcal{G}_i$ denotes the ith ($i \in \mathcal{I} = \{1, 2, ..., G\}$) multicast group user set. Assuming that each user only belongs to one multicast group, the number of multicast groups $G = |\mathcal{G}|$ satisfies $1 \le G \le K$. Also, let $G_i = |\mathcal{G}_i|$ be the number of users in the multi-cast group $\mathcal{G}_i$, then $\sum_{i=1}^{G} G_i = K$.

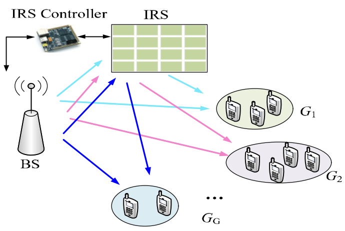

Fig. 1. IRS-assisted wireless multi-group multicast system.

Furthermore, define the reflection coefficient of the mth IRS reflection element as $\varphi_m = e^{j\theta_m}$, where $\theta_m \in [0, 2\pi), m = 1, 2, ..., M$, and denote $\mathbf{\Phi} = diag(\varphi_1, \varphi_2, ..., \varphi_M)$ as the diagonal matrix of IRS reflection coefficients. Besides, the channels between BS-IRS, IRS-user k, and BS-user k are defined as $\mathbf{G} \in \mathbb{C}^{M \times N}$, $\boldsymbol{h}_{r,k}^H \in \mathbb{C}^{1 \times M}$ and $\boldsymbol{h}_{d,k}^H \in \mathbb{C}^{1 \times N}$, respectively. Denote $\boldsymbol{w}_i \in \mathbb{C}^{N \times 1}$ as the beamforming vector of the BS to the multicast group $\mathcal{G}_i$. Then, the transmit signal of the BS is given by $\boldsymbol{x} = \sum_{i=1}^{G} \boldsymbol{w}_i s_i$, where s_i is the symbol transmitted from the BS to multicast group $\mathcal{G}_i$. Assume that $\{s_i\}_{i=1}^{G}$ are statistically irrelevant random variables with zero mean and unit variance, then $\mathbb{E}(s_i s_i^*) = 1$. Therefore, the signal received at the user k is

$$y_k = \left(\boldsymbol{h}_{r,k}^H \mathbf{\Phi} \mathbf{G} + \boldsymbol{h}_{d,k}^H\right) \sum_{i=1}^{G} \boldsymbol{w}_i s_i + n_k, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}, \quad (1)$$

where $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ represents the additive white Gaussian noise at the user k.

Define $\mathbf{H}_k = diag\left(\boldsymbol{h}_{r,k}^H\right) \mathbf{G}$ as the cascade channel established by the BS and the user k through the IRS, and $\boldsymbol{\varphi} = (\varphi_1, \varphi_2, ..., \varphi_M)^T$ as the IRS reflection coefficient vector. Then, (1) can be further represented by

$$y_k = \left(\boldsymbol{\varphi}^H \mathbf{H}_k + \boldsymbol{h}_{d,k}^H\right) \sum_{i=1}^{G} \boldsymbol{w}_i s_i + n_k, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}. \quad (2)$$

Therefore, the signal-to-interference-plus-noise ratio (SINR) for user k is given by

$$S_k = \frac{\left|\left(\boldsymbol{\varphi}^H \mathbf{H}_k + \boldsymbol{h}_{d,k}^H\right) \boldsymbol{w}_i\right|^2}{\sum_{j \neq i}^{G} \left|\left(\boldsymbol{\varphi}^H \mathbf{H}_k + \boldsymbol{h}_{d,k}^H\right) \boldsymbol{w}_j\right|^2 + \sigma_k^2}. \quad (3)$$

Furthermore, the CSI uncertainty models are as follows,

$$\begin{cases} \mathbf{H}_k = \bar{\mathbf{H}}_k + \Delta \mathbf{H}_k \\ \boldsymbol{h}_{d,k}^H = \bar{\boldsymbol{h}}_{d,k}^H + \Delta \boldsymbol{h}_{d,k}^H \end{cases}, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}, \quad (4)$$

in which $\bar{\mathbf{H}}_k$ and $\bar{\boldsymbol{h}}_{d,k}^H$ represent the estimated channels for the direct and cascade links, respectively, and $\Delta \mathbf{H}_k$ and $\Delta \boldsymbol{h}_{d,k}^H$ stand for the corresponding channel uncertainty errors, respectively. Herein, a kind of practical CSI error model, i.e., statistical CSI error [10], is considered, which is denoted by

$$vec(\Delta \mathbf{H}_k) \sim \mathcal{CN}(0, \mathbf{\Sigma}_{H,k}), \mathbf{\Sigma}_{H,k} \succeq \mathbf{0}, \quad (5a)$$

$$\Delta \boldsymbol{h}_{d,k}^H \sim \mathcal{CN}(0, \mathbf{\Sigma}_{d,k}), \mathbf{\Sigma}_{d,k} \succeq \mathbf{0}. \quad (5b)$$

where $\mathbf{\Sigma}_{H,k}$ and $\mathbf{\Sigma}_{d,k}$ represent the error covariance matrices.

III. Robust Optimization Design Problem

A. Fairness problem

First, the fairness problem under the statistical CSI error is discussed in this subsection, which is given by

$$\max_{\{\boldsymbol{w}_i\}_{i=1}^{G}, \mathbf{\Phi}} \min_{k \in \mathcal{G}_i, i \in \mathcal{I}} \Pr\{S_k/\gamma_k \ge R_k\} \quad (6a)$$

$$s.t. \sum_{i=1}^{G} \|\boldsymbol{w}_i\|_2^2 \le P, \quad (6b)$$

$$(4), (5a), (5b), \quad (6c)$$

$$|\varphi_m|^2 = 1, m = 1, 2, ..., M. \quad (6d)$$

where $\Pr\{\cdot\}$ represents the probability, R_k is the requirement of the weighted SINR and P is the maximum power at BS.

By introducing an auxiliary variable $t \in (0, 1]$, the optimization problem (6) can be equivalently denoted as

$$\max_{\{\boldsymbol{w}_i\}_{i=1}^{G}, \mathbf{\Phi}, t} t \quad (7a)$$

$$s.t. \sum_{i=1}^{G} \|\boldsymbol{w}_i\|_2^2 \le P, \quad (7b)$$

$$\Pr\{S_k/\gamma_k \ge R_k\} \ge t, \quad (7c)$$

$$(4), (5a), (5b), \quad (7d)$$

$$|\varphi_m|^2 = 1, m = 1, 2, ..., M. \quad (7e)$$

Since it is difficult to obtain the closed-form expression of the outage probability constraint (7c), the problem (7) is hard to deal with directly. Nevertheless, it can be tackled through the AO algorithm after transforming (7c) into a handleable form, whose specific procedure is as follows.

1) Constraint (7c) transformation: Before discussing the processing of (7c) in detail, the Bernstein-type inequality [11] is given as follows.

Lemma 1: Denote $\boldsymbol{e} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_n)$ as a circularly symmetric complex Gaussian (CSCG) random vector, $(\mathbf{Q}, \boldsymbol{r}, s) \in \mathbb{H}^n \times \mathbb{C}^n \times \mathbb{R}$ as arbitrary 3-tuple variables. For any $\rho \in [0,1]$, the following approximation holds:

$$\Pr\left\{\boldsymbol{e}^H\mathbf{Q}\boldsymbol{e} + 2\mathrm{Re}\left\{\boldsymbol{e}^H\boldsymbol{r}\right\} + s \geq 0\right\} \geq 1-\rho \tag{8a}$$

$$\begin{aligned}\Rightarrow & Tr(\mathbf{Q}) - \sqrt{2\ln(1/\rho)}\sqrt{\|\mathbf{Q}\|_F^2 + 2\|\boldsymbol{r}\|^2} \\ & + \ln(\rho)\lambda^+(-\mathbf{Q}) + s \geq 0\end{aligned} \tag{8b}$$

$$\Rightarrow \begin{cases} Tr(\mathbf{Q}) - \sqrt{2\ln(1/\rho)}x + \ln(\rho)y + s \geq 0 \\ \sqrt{\|\mathbf{Q}\|_F^2 + 2\|\boldsymbol{r}\|^2} \leq x \\ y\mathbf{I}_n + \mathbf{Q} \succeq \mathbf{0}, y \geq 0 \end{cases} \tag{8c}$$

where x and y are slack variables, $Tr\{\cdot\}$ denotes the trace of a matrix. $\lambda^+(-\mathbf{Q}) = \max(\lambda_{max}(-\mathbf{Q}), 0)$, in which $\lambda_{max}\{\cdot\}$ means the maximum eigenvalue of a matrix.

In order to utilize Lemma 1 to deal with (7c), let $\rho = 1-t$, then (7) can be further rewritten as

$$\min_{\{\boldsymbol{w}_i\}_{i=1}^G, \boldsymbol{\Phi}, \rho} \rho \tag{9a}$$

$$s.t.\ (7b), (7d), (7e), \tag{9b}$$

$$\begin{aligned}&\Pr\left\{\left(\boldsymbol{\varphi}^H\mathbf{H}_k + \boldsymbol{h}_{d,k}^H\right)\boldsymbol{\Theta}_k\left(\boldsymbol{\varphi}^H\mathbf{H}_k + \boldsymbol{h}_{d,k}^H\right)^H - \sigma_k^2 \geq 0\right\} \\ &\geq 1-\rho, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I},\end{aligned} \tag{9c}$$

where $\boldsymbol{\Theta}_k = \boldsymbol{w}_i\boldsymbol{w}_i^H/\gamma_k R_k - \mathbf{W}_{-i}\mathbf{W}_{-i}^H$, $\mathbf{W}_{-i} \triangleq (\boldsymbol{w}_1, \boldsymbol{w}_2, ..., \boldsymbol{w}_{i-1}, \boldsymbol{w}_{i+1}, ..., \boldsymbol{w}_G)$.

According to [12], assuming $\boldsymbol{\Sigma}_{H,k} = \varepsilon_{H,k}^2\mathbf{I}$ and $\boldsymbol{\Sigma}_{d,k} = \varepsilon_{d,k}^2\mathbf{I}$, the statistical CSI error model (5) can be simplified as

$$vec(\Delta\mathbf{H}_k) = \varepsilon_{H,k}\boldsymbol{i}_{H,k}, \boldsymbol{i}_{H,k} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}), \tag{10a}$$

$$\Delta\boldsymbol{h}_k = \varepsilon_{d,k}\boldsymbol{i}_{d,k}, \boldsymbol{i}_{d,k} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}), \tag{10b}$$

Then, the outage probability in (9c) can be rewritten as

$$\begin{aligned}&\Pr\left\{\left(\boldsymbol{\varphi}^H\bar{\mathbf{H}}_k + \bar{\boldsymbol{h}}_{d,k}^H\right)\boldsymbol{\Theta}_k\left(\bar{\mathbf{H}}_k^H\boldsymbol{\varphi} + \bar{\boldsymbol{h}}_{d,k}\right) - \sigma_k^2\right. \\ &+ 2\mathrm{Re}\left\{\left(\boldsymbol{\varphi}^H\bar{\mathbf{H}}_k + \bar{\boldsymbol{h}}_{d,k}^H\right)\boldsymbol{\Theta}_k\left(\Delta\mathbf{H}_k^H\boldsymbol{\varphi} + \Delta\mathbf{h}_{d,k}\right)\right\} \\ &\left. + \left(\boldsymbol{\varphi}^H\Delta\mathbf{H}_k + \Delta\boldsymbol{h}_{d,k}^H\right)\boldsymbol{\Theta}_k\left(\Delta\mathbf{H}_k^H\boldsymbol{\varphi} + \Delta\boldsymbol{h}_{d,k}\right) \geq 0\right\}.\end{aligned} \tag{11}$$

The third term of (11) can be further denoted by

$$\begin{aligned}&2\mathrm{Re}\left\{\varepsilon_{d,k}\left(\boldsymbol{\varphi}^H\bar{\mathbf{H}}_k + \bar{\boldsymbol{h}}_{d,k}^H\right)\boldsymbol{\Theta}_k\boldsymbol{i}_{d,k}\right. \\ &\left.+\varepsilon_{H,k}vec^T\left(\boldsymbol{\varphi}\left(\boldsymbol{\varphi}^H\bar{\mathbf{H}}_k + \bar{\boldsymbol{h}}_{d,k}^H\right)\boldsymbol{\Theta}_k\right)\boldsymbol{i}_{H,k}^*\right\} \\ &\triangleq 2\mathrm{Re}\left\{\boldsymbol{u}_k^H\boldsymbol{i}_k\right\},\end{aligned} \tag{12}$$

in which $\boldsymbol{i}_k = \left[\boldsymbol{i}_{d,k}^H, \boldsymbol{i}_{H,k}^T\right]^H$, $\boldsymbol{u}_k = \begin{bmatrix} \varepsilon_{d,k}\boldsymbol{\Theta}_k\left(\bar{\mathbf{H}}_k^H\boldsymbol{\varphi} + \bar{\boldsymbol{h}}_{d,k}\right) \\ \varepsilon_{H,k}vec^*\left(\boldsymbol{\varphi}\left(\boldsymbol{\varphi}^H\bar{\mathbf{H}}_k + \bar{\boldsymbol{h}}_{d,k}^H\right)\boldsymbol{\Theta}_k\right)\end{bmatrix}$.

Besides, the fourth term of (11) can be further denoted by

$$\begin{aligned}&\varepsilon_{d,k}^2\boldsymbol{i}_{d,k}^H\boldsymbol{\Theta}_k\boldsymbol{i}_{d,k} + 2\mathrm{Re}\left\{\varepsilon_{d,k}\varepsilon_{H,k}\boldsymbol{i}_{d,k}^H\left(\boldsymbol{\Theta}_k \otimes \boldsymbol{\varphi}^T\right)\boldsymbol{i}_{H,k}^*\right\} \\ &+ \varepsilon_{H,k}^2\boldsymbol{i}_{H,k}^T\left(\boldsymbol{\Theta}_k \otimes \boldsymbol{\Xi}^T\right)\boldsymbol{i}_{H,k}^* \\ &\triangleq \boldsymbol{i}_k^H\mathbf{U}_k\boldsymbol{i}_k,\end{aligned} \tag{13}$$

in which $\otimes$ is the Kronecker product, $\boldsymbol{\Xi} = \boldsymbol{\varphi}\boldsymbol{\varphi}^H$, $\mathbf{U}_k = \begin{bmatrix} \varepsilon_{d,k}^2\boldsymbol{\Theta}_k & \varepsilon_{d,k}\varepsilon_{H,k}\left(\boldsymbol{\Theta}_k \otimes \boldsymbol{\varphi}^T\right) \\ \varepsilon_{d,k}\varepsilon_{H,k}\left(\boldsymbol{\Theta}_k \otimes \boldsymbol{\varphi}^*\right) & \varepsilon_{H,k}^2\left(\boldsymbol{\Theta}_k \otimes \boldsymbol{\Xi}^T\right)\end{bmatrix}$.

Substituting (12) and (13) into (11), the outage probability constraint can be equivalently given by

$$\Pr\left(\boldsymbol{i}_k^H\mathbf{U}_k\boldsymbol{i}_k + 2\mathrm{Re}\left\{\boldsymbol{u}_k^H\boldsymbol{i}_k\right\} + u_k \geq 0\right) \geq 1-\rho, \tag{14}$$

where $u_k = \left(\boldsymbol{\varphi}^H\bar{\mathbf{H}}_k + \bar{\boldsymbol{h}}_{d,k}^H\right)\boldsymbol{\Theta}_k\left(\bar{\mathbf{H}}_k^H\boldsymbol{\varphi} + \bar{\boldsymbol{h}}_{d,k}\right) - \sigma_k^2$.

Then, by introducing auxiliary variables $\boldsymbol{x} = [x_1, ..., x_K]^T$ and adopting Lemma 1, the data outage probability of user k can be approximated as

$$Tr(\mathbf{U}_k) - \sqrt{2\ln(1/\rho)}x_k + \ln(\rho)y_k + u_k \geq 0, \tag{15a}$$

$$\sqrt{\|\mathbf{U}_k\|_F^2 + 2\|\boldsymbol{u}_k\|^2} \leq x_k, \tag{15b}$$

$$y_k\mathbf{I} + \mathbf{U}_k \succeq \mathbf{0}, y_k \geq 0. \tag{15c}$$

The terms in (15) can be simplified as

$$Tr(\mathbf{U}_k) = \left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2M\right)Tr(\boldsymbol{\Theta}_k), \tag{16a}$$

$$\|\mathbf{U}_k\|_F^2 = \left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2M\right)^2\|\boldsymbol{\Theta}_k\|_F^2, \tag{16b}$$

$$\|\boldsymbol{u}_k\|^2 = \left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2M\right)\left\|\left(\boldsymbol{\varphi}^H\bar{\mathbf{H}}_k + \bar{\boldsymbol{h}}_{d,k}^H\right)\boldsymbol{\Theta}_k\right\|_2^2, \tag{16c}$$

$$y_k\mathbf{I} + \mathbf{U}_k \succeq \mathbf{0} \Rightarrow y_k\mathbf{I} + \left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2M\right)\boldsymbol{\Theta}_k \succeq \mathbf{0}. \tag{16d}$$

Finally, the problem (10) can be equivalently denoted by

$$\min_{\boldsymbol{w}_i, \boldsymbol{\varphi}, \rho, \boldsymbol{x}, \boldsymbol{y}} \rho \tag{17a}$$

$$s.t.\ (7b), (7e), \tag{17b}$$

$$\begin{aligned}&\left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2M\right)Tr(\boldsymbol{\Theta}_k) - \sqrt{2\ln(1/\rho)}x_k + \ln(\rho)y_k \\ &+ u_k \geq 0, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I},\end{aligned} \tag{17c}$$

$$\begin{aligned}&\left\|\begin{matrix}\left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2M\right)vec(\boldsymbol{\Theta}_k) \\ \sqrt{2\left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2M\right)}\boldsymbol{\Theta}_k\left(\bar{\mathbf{H}}_k^H\boldsymbol{\varphi} + \bar{\boldsymbol{h}}_{d,k}\right)\end{matrix}\right\| \leq x_k, \\ &\forall k \in \mathcal{G}_i, \forall i \in \mathcal{I},\end{aligned} \tag{17d}$$

$$y_k\mathbf{I} + \left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2M\right)\boldsymbol{\Theta}_k \succeq \mathbf{0}, y_k \geq 0, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}. \tag{17e}$$

Due to the unit modulus constraint and the coupling of the optimization variables $\boldsymbol{w}_i$ and $\boldsymbol{\varphi}$, the optimization problem (17) is still non-convex and intractable. Thus, an AO based algorithm is designed to solve it iteratively.

2) Optimization of the transmit beamforming and auxiliary variables: For the given IRS reflection coefficient $\boldsymbol{\varphi}$, the transmit beamforming and auxiliary variables are optimized herein. Define $\mathbf{T}_i = \boldsymbol{w}_i\boldsymbol{w}_i^H$, then $\boldsymbol{\Theta}_k = \mathbf{T}_i/\gamma_k R_k - \sum_{j\neq i}^G \mathbf{T}_j$. The subproblem is given by

$$\min_{\mathbf{T}_i, \rho, \boldsymbol{x}, \boldsymbol{y}} \rho \tag{18a}$$

$$s.t.\ (17c), (17d), (17e), \tag{18b}$$

$$\sum_{i=1}^G Tr(\mathbf{T}_i) \leq P, \mathbf{T}_i \succeq \mathbf{0}, rank(\mathbf{T}_i) = 1, \forall i \in \mathcal{I}. \tag{18c}$$

Herein, (18) can be solved by performing a bisection search on ρ and solving a series of feasibility-check problems [13].

However, due to the rank-1 constraint, feasibility-check problems are still non-convex. By removing the rank-1 constraint through the semi-definite relaxation (SDR) method [14], they become convex problems, which can be solved by the CVX tool. Then the obtained $\mathbf{T}_i$ can be further processed through the Gaussian randomization method [14] to derive the transmit beamforming.

3) Optimization of IRS reflection coefficients and auxiliary variables: Now, for the fixed transmit beamforming $\boldsymbol{w}_i$, IRS reflection coefficients and auxiliary variables are optimized. The subproblem is given by

$$\min_{\boldsymbol{\varphi},\rho,\boldsymbol{x},\boldsymbol{y}} \rho \tag{19a}$$

$$s.t.\ (7e), (17c), (17d), (17e). \tag{19b}$$

Similar to the optimization problem (18), (19) can be solved by performing a bisection search on ρ and solving a series of feasibility-check problems. According to [15], in order to improve the convergence of $\boldsymbol{\varphi}$, slack variables $\boldsymbol{\beta} = (\beta_1, ..., \beta_K)^T \geq 0$ are introduced herein, and u_k in the constraint (17c) is updated by

$$\begin{aligned} u_k' &= \left(\boldsymbol{\varphi}^H \bar{\mathbf{H}}_k + \bar{\boldsymbol{h}}_{d,k}^H\right) \boldsymbol{\Theta}_k \left(\bar{\mathbf{H}}_k^H \boldsymbol{\varphi} + \bar{\boldsymbol{h}}_{d,k}\right) - \sigma_k^2 - \beta_k \\ &= Tr\left(\mathbf{P}_k \mathbf{E}\right) + \bar{\boldsymbol{h}}_{d,k}^H \boldsymbol{\Theta}_k \bar{\boldsymbol{h}}_{d,k} - \sigma_k^2 - \beta_k, \end{aligned} \tag{20}$$

where $\mathbf{P}_k = \begin{bmatrix} \bar{\mathbf{H}}_k \boldsymbol{\Theta}_k \bar{\mathbf{H}}_k^H & \bar{\mathbf{H}}_k \boldsymbol{\Theta}_k \bar{\boldsymbol{h}}_{d,k} \\ \bar{\boldsymbol{h}}_{d,k}^H \boldsymbol{\Theta}_k \bar{\mathbf{H}}_k^H & 0 \end{bmatrix}$, $\mathbf{E} = \boldsymbol{e}\boldsymbol{e}^H$, $\boldsymbol{e} = \begin{bmatrix} \boldsymbol{\varphi} & 1 \end{bmatrix}^T$.

Furthermore, the constraint (17d) can be rewritten as

$$\begin{aligned} &\left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2 M\right)^2 \left\|\boldsymbol{\Theta}_k\right\|_F^2 \\ &+ 2\left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2 M\right)\left(Tr\left(\mathbf{Q}_k \mathbf{E}\right) + \bar{\boldsymbol{h}}_{d,k}^H \boldsymbol{\Theta}_k^H \boldsymbol{\Theta}_k \bar{\boldsymbol{h}}_{d,k}\right) \\ \leq\ & 2\mathrm{Re}\left\{x_k^{(n)} x_k\right\} - \left(x_k^{(n)}\right)^2, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}, \end{aligned} \tag{21}$$

in which $\mathbf{Q}_k = \begin{bmatrix} \bar{\mathbf{H}}_k \boldsymbol{\Theta}_k^H \boldsymbol{\Theta}_k \bar{\mathbf{H}}_k^H & \bar{\mathbf{H}}_k \boldsymbol{\Theta}_k^H \boldsymbol{\Theta}_k \bar{\boldsymbol{h}}_{d,k} \\ \bar{\boldsymbol{h}}_{d,k}^H \boldsymbol{\Theta}_k^H \boldsymbol{\Theta}_k \bar{\mathbf{H}}_k^H & 0 \end{bmatrix}$, $x_k^{(n)}$ is the optimal solution in the nth iteration.

Note that the constraint (17e) is independent of $\boldsymbol{\varphi}$, and it is transformed from $\lambda^+(-\mathbf{Q})$ in Lemma 1, so we have $y_k = \max\left(\lambda_{max}\left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2 M\right)\boldsymbol{\Theta}_k, 0\right), \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}$.

Therefore, the feasibility-check problem about $\boldsymbol{\varphi}$ corresponding to the problem (19) can be formulated as

$$\max_{\mathbf{E},\boldsymbol{x},\boldsymbol{y}} \sum_{k=1}^{K} \beta_k \tag{22a}$$

$$\begin{aligned} s.t.\ & \left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2 M\right) Tr\left(\boldsymbol{\Theta}_k\right) - \sqrt{2\ln\left(1/\rho_k\right)} x_k \\ & + \ln\left(\rho_k\right) y_k + u_k' \geq 0, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}, \end{aligned} \tag{22b}$$

$$(21), \boldsymbol{\beta} \geq 0, \tag{22c}$$

$$\mathbf{E} \succeq \mathbf{0}, rank\left(\mathbf{E}\right) = 1, [\mathbf{E}]_{m,m} = 1, m = 1, ..., M. \tag{22d}$$

Still, (22) is non-convex due to the rank-1 constraint in (22d), which can be removed through the SDR method [14]. The resulting convex problem can be solved by the CVX tool. If the obtained optimal solution $\mathbf{E}^\star$ satisfies the rank-1 constraint, the solution to (19) is derived by $\mathbf{E}^\star = \boldsymbol{e}^\star \boldsymbol{e}^{\star,H}$; Otherwise, the desired IRS reflection coefficient $\boldsymbol{\varphi}$ is derived through the Gaussian randomization method [14].

In summary, based on the AO algorithm, the problem (6) can be solved by alternately optimizing the transmit beamforming $\boldsymbol{w}_i$ and the IRS reflection coefficient $\boldsymbol{\varphi}$.

Besides, the approximate complexity of the subproblem (18) is $o_F = \mathcal{O}\left(\left[K\left(2N+2\right)\right]^{1/2} n_1 \left(n_1^2 + 2n_1 K N^2 + 2KN^3 + n_1 K \left(N^2+N\right)^2\right)\right)$, where $n_1 = GN$, and that of the subproblem (22) is $o_e = \mathcal{O}\left(M^{1/2} n_2 \left(n_2^2 + n_2 M^2 + M^3\right)\right)$, where $n_2 = M$. Then, the approximate complexity of the problem (6) is $o_F + o_e$.

B. QoS problem

The QoS problem under the statistical CSI error model is further discussed in this subsection. The transmit beamforming and the IRS reflection coefficient are jointly optimized to minimize the transmit power while satisfying the outage probability of users. Specifically, the optimization problem is as below

$$\min_{\{\boldsymbol{w}_i\}_{i=1}^G, \boldsymbol{\Phi}} \sum_{i=1}^{G} \left\|\boldsymbol{w}_i\right\|_2^2 \tag{23a}$$

$$\begin{aligned} s.t.\ & \Pr\left\{\log_2\left(1+S_k\right) \geq r_k\right\} \geq 1-\rho_k, \\ & \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}, \end{aligned} \tag{23b}$$

$$(4), (5a), (5b), \tag{23c}$$

$$\left|\varphi_m\right|^2 = 1, m = 1, 2, ..., M, \tag{23d}$$

where $\rho_k \in (0, 1]$ represents the outage probability. Therefore, constraint (23b) guarantees that the probability of each user's communication rate greater than r_k is not less than $1-\rho_k$. Similarly to that for constraint (7c), (23b) is first rewritten as

$$\begin{aligned} & \Pr\left\{\left(\boldsymbol{\varphi}^H \mathbf{H}_k + \boldsymbol{h}_{d,k}^H\right) \boldsymbol{\Theta}'_k \left(\boldsymbol{\varphi}^H \mathbf{H}_k + \boldsymbol{h}_{d,k}^H\right)^H - \sigma_k^2 \geq 0\right\} \\ \geq & 1-\rho_k, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}, \end{aligned} \tag{24}$$

in which $\boldsymbol{\Theta}'_k = \boldsymbol{w}_i \boldsymbol{w}_i^H / (2^{r_k} - 1) - \mathbf{W}_{-i} \mathbf{W}_{-i}^H$.

Furthermore, utilizing the same method as (9), the equivalent optimization problem of (23) can be obtained as

$$\min_{\boldsymbol{w}_i,\boldsymbol{\varphi},\boldsymbol{x},\boldsymbol{y}} \sum_{i=1}^{G} \left\|\boldsymbol{w}_i\right\|_2^2 \tag{25a}$$

$$\begin{aligned} s.t.\ & \left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2 M\right) Tr\left(\boldsymbol{\Theta}'_k\right) - \sqrt{2\ln\left(1/\rho_k\right)} x_k \\ & + \ln\left(\rho_k\right) y_k + u_k'' \geq 0, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}, \end{aligned} \tag{25b}$$

$$\begin{aligned} & \left\| \begin{matrix} \left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2 M\right) vec\left(\boldsymbol{\Theta}'_k\right) \\ \sqrt{2\left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2 M\right)} \boldsymbol{\Theta}'_k \left(\bar{\mathbf{H}}_k^H \boldsymbol{\varphi} + \bar{\boldsymbol{h}}_{d,k}\right) \end{matrix} \right\| \leq x_k, \\ & \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}, \end{aligned} \tag{25c}$$

$$y_k \mathbf{I} + \left(\varepsilon_{d,k}^2 + \varepsilon_{H,k}^2 M\right) \boldsymbol{\Theta}'_k \succeq \mathbf{0}, y_k \geq 0, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}, \tag{25d}$$

$$\left|\varphi_m\right|^2 = 1, m = 1, 2, ..., M, \tag{25e}$$

where $u_k'' = \left(\boldsymbol{\varphi}^H \bar{\mathbf{H}}_k + \bar{\boldsymbol{h}}_{d,k}^H\right) \boldsymbol{\Theta}'_k \left(\bar{\mathbf{H}}_k^H \boldsymbol{\varphi} + \bar{\boldsymbol{h}}_{d,k}\right) - \sigma_k^2$.

However, the optimization problem (25) is still non-convex and intractable due to the coupling of the optimization variables

$\boldsymbol{w}_i$ and $\boldsymbol{\varphi}$. Nevertheless, it can be solved by the AO method, and the specific process is as follows.

First, for the given IRS reflection coefficient $\boldsymbol{\varphi}$, the transmit beamforming $\boldsymbol{w}_i$ is optimized. Defining $\mathbf{T}_i = \boldsymbol{w}_i \boldsymbol{w}_i^H$, $\boldsymbol{\Theta'}_k = \mathbf{T}_i \big/ (2^{r_k} - 1) - \sum_{j \neq i}^{G} \mathbf{T}_j$, the subproblem is given by

$$\min_{\mathbf{T}_i, \boldsymbol{x}, \boldsymbol{y}} \sum_{i=1}^{G} Tr\left(\mathbf{T}_i\right) \tag{26a}$$

$$s.t.\ \ (25b)\,, (25c)\,, (25d)\,, \tag{26b}$$

$$\mathbf{T}_i \succeq \mathbf{0}, rank\left(\mathbf{T}_i\right) = 1, \forall i \in \mathcal{I}, \tag{26c}$$

Due to the rank-1 constraint, (26) is non-convex. Similarly, the SDR method is adopted to remove the rank-1 constraint, and the resulting optimization problem can be solved by the CVX tool. Then, the Gaussian randomization method is further applied to obtain the beamforming vector.

Next, for the given transmit beamforming $\boldsymbol{w}_i$, the IRS reflection coefficient $\boldsymbol{\varphi}$ is optimized. The optimization subproblem becomes a feasibility-check problem about $\boldsymbol{\varphi}$, which is the same as (19) when fixing ρ.

Finally, the approximate complexity of the problem (23) is consistent with that of (6), which is omitted here. More details about the proposed algorithm can be found in [16].

IV. SIMULATION RESULTS

Numerical simulations are presented in this section to evaluate the performance of the proposed algorithm. Specifically, the antenna of the BS is $N = 4$, the number of multicast groups is $G = 3$, the total number of users is $K = 6$, the number of IRS reflection elements is $M = 20$, the average noise power of the users is set to $\sigma_k^2 = -80dBm$, and the transmit power is set as $P = 36dBm$. The coordinates of the BS and the IRS are (0,0) and (40,8) respectively. Besides, the users are randomly distributed in their respective multicast groups, whose abscissa range is (60,75) and ordinate range is (-5,5). The channel between any two nodes can be denoted as $\mathbf{H} = \sqrt{L_0 (d/d_0)^{-\alpha}} \mathbf{G}_{ssf}$, where L_0 represents the path loss at the reference distance $d_0 = 1m$, d is the distance between two nodes and α means the path loss exponent. $\mathbf{G}_{ssf}$ stands for the small scale fading component, which is given by $\mathbf{G}_{ssf} = \sqrt{\beta / (1+\beta)} \mathbf{G}_{ssf}^{LoS} + \sqrt{1/(1+\beta)} \mathbf{G}_{ssf}^{NLoS}$, where β is the Rician factor, $\mathbf{G}^{LoS}$ and $\mathbf{G}^{NLoS}$ denote light-of-sight (LoS) components and non-LoS components, respectively. Herein, Rician factors are set as 5 and the path loss exponents are set as $\alpha_{BU} = 3.6$, $\alpha_{BI} = 2.2$ and $\alpha_{IU} = 2$.

For the statistical CSI error model, the variances of $vec\left(\Delta \mathbf{H}_k\right)$ and $\Delta \boldsymbol{h}_k$ are defined as $\varepsilon_{H,k}^2 = \delta_H^2 \left\| vec\left(\bar{\mathbf{H}}_k\right) \right\|^2$ and $\varepsilon_{d,k}^2 = \delta_d^2 \left\| \bar{\boldsymbol{h}}_k \right\|^2$, respectively, where δ_H and δ_d stand for the relative amount of CSI uncertainties. Besides, the outage probability is set as $\rho_k = 0.05$.

In addition, two comparison schemes are introduced here. The first one is the non-IRS scheme, in which the design is only based on the transmit beamforming [17]. The other one is the discrete scheme which is based on the proposed AO algorithm but the IRS has discrete phase shifts [18]. Besides, for the discrete scheme, only 2-bit quantization is considered.

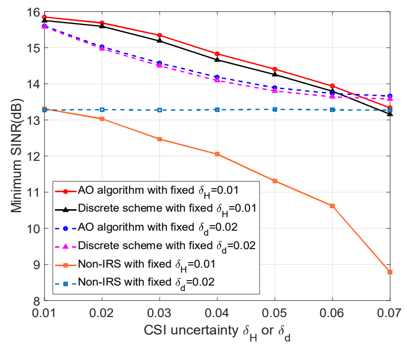

Fig. 2. The influence of CSI uncertainty on the minimum SINR for the fairness problem

For the fairness problem, the influence of the channel uncertainty on the minimum SINR of users is analyzed, and the results are shown in Fig. 2. First, fix the cascade channel uncertainty as $\delta_H = 0.01$ and increase the direct link channel uncertainty δ_d, whose results are shown as the solid lines. Although the performance of all algorithms is degraded, the SINR improvement brought by the proposed AO algorithm and the discrete scheme is clearly better than that of the non-IRS scheme. Subsequently, the direct link channel uncertainty is fixed as $\delta_d = 0.02$ and the cascade channel uncertainty δ_H is gradually increased, whose results are shown as the dotted lines. The minimum SINR obtained by the AO algorithm and the discrete scheme gradually decreases and tends to the performance of the non-IRS scheme, while the non-IRS scheme is not affected, since there is no cascade channel. Besides, the discrete scheme is always close to the performance of the AO algorithm, whose gap in minimum SINR is less than 0.2 dB. By fixing one channel uncertainty δ_d (or δ_H) unchanged and observing the influence of the other one, it can be seen that the direct link channel uncertainty has a greater impact on the proposed algorithm and the discrete scheme, whose minimum SINR experiences a faster decrease.

Then, for the QoS problem, the influence of the channel uncertainty on the BS transmit power is analyzed, and the results are shown in Fig. 3. It is assumed that the minimum communication rate for all users is set as $r_k = 3$ bit/s/Hz. First, fix the cascade channel uncertainty as $\delta_H = 0.01$ and increase the direct link channel uncertainty δ_d, whose results are shown as the solid lines. All the schemes require more transmit power, among which the proposed AO algorithm and the discrete scheme can significantly save transmit power budgets. Then, the direct link channel uncertainty is fixed as $\delta_d = 0.02$ and the cascade channel uncertainty δ_H is gradually increased, whose results are shown as the dotted lines. The non-IRS scheme is not affected, but the transmit power consumption of the AO algorithm and discrete scheme are getting higher. Similarly, the discrete scheme is always close to the performance of the AO algorithm, whose gap in

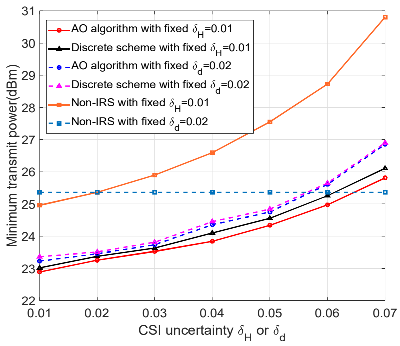

Fig. 3. The influence of CSI uncertainty on the transmit power for the QoS problem

power budget is less than 0.3 dBm. By fixing one channel uncertainty δ_d (or δ_H) unchanged and observing the influence of the other one, it can be seen that the cascade channel uncertainty has a greater impact on the proposed algorithm, leading to more transmit power consumption and a faster growth trend. Moreover, when $\delta_d = 0.02$ and $\delta_H > 0.055$, the power consumption of the AO algorithm is higher than that of the non-IRS scheme, which indicates that the proposed scheme is not useful anymore.

V. CONCLUSION

The robust design of IRS-assisted wireless multi-group multicast system is discussed in this paper, where the fairness problem and the QoS problem under the statistical CSI error model are considered. The Bernstein-type inequality is utilized to transform the outage probability into the SOC constraint and linear inequalities. Then, an efficient AO based algorithm is proposed to solve the formulated non-convex problem, where the feasibility-check method, SDR method and Gaussian randomization are adopted. Simulation results indicate that the proposed algorithm can effectively improve the performance of multi-group multicast systems. However, the performance gain obtained by the proposed algorithm can be significantly affected by the channel uncertainty error level. It means that the precise channel estimation is crucial for the IRS-assisted communication system, otherwise deploying IRS will not bring performance gains to the system, or even worsen the system performance.

REFERENCES

[1] Q. Wu and R. Zhang, "Towards Smart and Reconfigurable Environment: Intelligent Reflecting Surface Aided Wireless Network," *IEEE Commun. Mag.*, vol. 58, no. 1, pp. 106–112, 2020.

[2] Y. Omid, S. M. Shahabi, C. Pan, Y. Deng, and A. Nallanathan, "Low-Complexity Robust Beamforming Design for IRS-Aided MISO Systems With Imperfect Channels," *IEEE Commun. Lett.*, vol. 25, no. 5, pp. 1697–1701, 2021.

[3] M.-M. Zhao, A. Liu, and R. Zhang, "Outage-Constrained Robust Beamforming for Intelligent Reflecting Surface Aided Wireless Communication," *IEEE Trans. Signal Process.*, vol. 69, pp. 1301–1316, 2021.

[4] L. Zhang, C. Pan, Y. Wang, H. Ren, and K. Wang, "Robust Beamforming Design for Intelligent Reflecting Surface Aided Cognitive Radio Systems With Imperfect Cascaded CSI," *IEEE Trans. on Cogn. Commun. Netw.*, vol. 8, no. 1, pp. 186–201, 2022.

[5] G. Zhou, C. Pan, H. Ren, K. Wang, M. D. Renzo, and A. Nallanathan, "Robust Beamforming Design for Intelligent Reflecting Surface Aided MISO Communication Systems," *IEEE Wireless Commun. Lett.*, vol. 9, no. 10, pp. 1658–1662, 2020.

[6] X. Yu, D. Xu, D. W. K. Ng, and R. Schober, "IRS-Assisted Green Communication Systems: Provable Convergence and Robust Optimization," *IEEE Trans. Commun.*, vol. 69, no. 9, pp. 6313–6329, 2021.

[7] G. Zhou, C. Pan, H. Ren, K. Wang, and A. Nallanathan, "Intelligent Reflecting Surface Aided Multigroup Multicast MISO Communication Systems," *IEEE Trans. Signal Process.*, vol. 68, pp. 3236–3251, 2020.

[8] W. Shi, J. Li, G. Xia, Y. Wang, X. Zhou, Y. Zhang, and F. Shu, "Secure Multigroup Multicast Communication Systems via Intelligent Reflecting Surface," *China Commun.*, vol. 18, no. 3, pp. 39–51, 2021.

[9] Q. Tao, S. Zhang, C. Zhong, and R. Zhang, "Intelligent Reflecting Surface Aided Multicasting With Random Passive Beamforming," *IEEE Wireless Commun. Lett.*, vol. 10, no. 1, pp. 92–96, 2021.

[10] J. Zhang, M. Kountouris, J. G. Andrews, and R. W. Heath, "Multi-Mode Transmission for the MIMO Broadcast Channel with Imperfect Channel State Information," *IEEE Trans. Commun.*, vol. 59, no. 3, pp. 803–814, 2011.

[11] K.-Y. Wang, A. M.-C. So, T.-H. Chang, W.-K. Ma, and C.-Y. Chi, "Outage Constrained Robust Transmit Optimization for Multiuser MISO Downlinks: Tractable Approximations by Conic Optimization," *IEEE Trans. Signal Process.*, vol. 62, no. 21, pp. 5690–5705, 2014.

[12] G. Zhou, C. Pan, H. Ren, K. Wang, and A. Nallanathan, "A Framework of Robust Transmission Design for IRS-Aided MISO Communications With Imperfect Cascaded Channels," *IEEE Trans. Signal Process.*, vol. 68, pp. 5092–5106, 2020.

[13] C. Kai, W. Ding, and W. Huang, "Max-Min Fairness in IRS-Aided MISO Broadcast Channel via Joint Transmit and Reflective Beamforming," in *IEEE GLOBECOM*, 2020, pp. 1–6.

[14] Z.-q. Luo, W.-k. Ma, A. M.-c. So, Y. Ye, and S. Zhang, "Semidefinite Relaxation of Quadratic Optimization Problems," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, 2010.

[15] Q. Wu and R. Zhang, "Intelligent Reflecting Surface Enhanced Wireless Network via Joint Active and Passive Beamforming," *IEEE Trans. Wireless Commun.*, vol. 18, no. 11, pp. 5394–5409, 2019.

[16] W. Jiang, P. Xiong, J. Nie, Z. Ding, C. Pan, and Z. Xiong, "Robust Design of IRS-Aided Multi-group Multicast System with Imperfect CSI," *To be submitted to IEEE Trans. Wireless Commun.*, 2022.

[17] N. Sidiropoulos, T. Davidson, and Z.-Q. Luo, "Transmit Beamforming for Physical-Layer Multicasting," *IEEE Trans. Signal Process.*, vol. 54, no. 6, pp. 2239–2251, 2006.

[18] W. Jiang, Y. Zhang, J. Wu, W. Feng, and Y. Jin, "Intelligent Reflecting Surface Assisted Secure Wireless Communications With Multiple-Transmit and Multiple-Receive Antennas," *IEEE Access*, vol. 8, pp. 86 659–86 673, 2020.