Assignment 5

(1) Find the error in the following "proof" of "For any positive integer n, a connected graph with n vertices has at least $n - 1$ edges." The error is not a typographical error nor a error in wording. It is an error in the argument. Explain your answer.

Proof. We prove the statement "A connected graph with n vertices has at least $n - 1$ edges," for $n = 1, 2, 3, \ldots$

Basis: A connected graph with one vertex clearly has at least 0 edges.

Induction Step: Assume n is a positive integer and any connected graph with n vertices has at least $n - 1$ edges. Prove a connected graph with $n + 1$ vertices has at least n edges. Let G_n be a connected graph with n vertices. The induction hypothesis implies that G_n has at least $n - 1$ edges. Add a vertex to G_n and an edge from this new vertex to one of the vertices in G_n to obtain a connected graph, which we will call G_{n+1}, with $n + 1$ vertices and at least n edges. This completes the induction step, so by the principal of mathematical induction a connected graph with n vertices has at least $n - 1$ edges for any positive integer n.

$\square$

(2) Prove a graph with n vertices and at least n edges contains a cycle for all positive integers n. You may use that a graph with all vertices of degree at least 2 contains a cycle. (Use the [Rosen] definition of *graph*.)