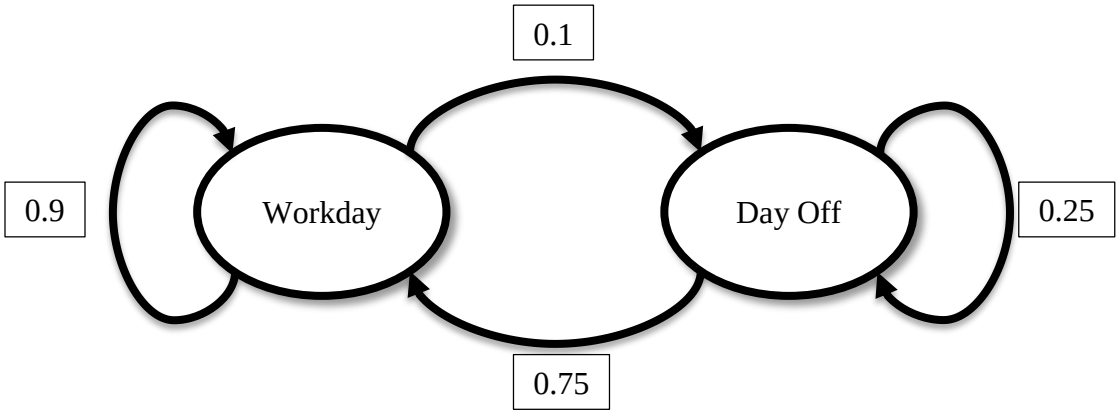
Name:
Course: CAP 4601
Semester: Summer 2013
Assignment: Assignment 08
Date: 24 JUL 2013

Complete the following written problems:

1. Markov Chain (260 Points).

Given the following Markov Chain diagram:



Given that the first day was a workday (i.e. $P(Day\,1 = Workday) = 1$ and $P(Day\,1 = Day\ Off) = 0$), what is the following:

a. (20 Points) What is the probability that the second day is a Workday (i.e. $P(Day\,2 = Workday)$)?

$$P(Day\,2 = Workday) = \overbrace{\begin{pmatrix} P(Day\,2 = Workday \mid Day\,1 = Workday)\,P(Day\,1 = Workday) \\ + \\ P(Day\,2 = Workday \mid Day\,1 = Day\ Off)\,P(Day\,1 = Day\ Off) \end{pmatrix}}^{\text{Total Probability}}$$

$$= \begin{pmatrix} (0.9)(1) \\ + \\ (0.75)(0) \end{pmatrix}$$

$$= 0.9$$

b. (20 Points) What is the probability that the second day was a Day Off (i.e. $P(Day\ 2 = Day\ Off)$)?

$$P(Day\ 2 = Day\ Off) = \overbrace{\begin{pmatrix} P(Day\ 2 = Day\ Off \mid Day\ 1 = Workday)P(Day\ 1 = Workday) \\ + \\ P(Day\ 2 = Day\ Off \mid Day\ 1 = Day\ Off)P(Day\ 1 = Day\ Off) \end{pmatrix}}^{\text{Total Probability}}$$

$$= \begin{pmatrix} (0.1)(1) \\ + \\ (0.25)(0) \end{pmatrix}$$

$$= 0.1$$

c. (40 Points) What is the probability that the third day was a Workday (i.e. $P(Day\ 3 = Workday)$)?

$$P(Day\ 3 = Workday) = \overbrace{\begin{pmatrix} P(Day\ 3 = Workday \mid Day\ 2 = Workday)P(Day\ 2 = Workday) \\ + \\ P(Day\ 3 = Workday \mid Day\ 2 = Day\ Off)P(Day\ 2 = Day\ Off) \end{pmatrix}}^{\text{Total Probability}}$$

$$= \begin{pmatrix} (0.9)(0.9) \\ + \\ (0.75)(0.1) \end{pmatrix}$$

$$= 0.885$$

d. (40 Points) What is the probability that the third day was a Day Off (i.e. $P(Day\ 3 = Day\ Off)$)?

$$P(Day\ 3 = Day\ Off) = \overbrace{\begin{pmatrix} P(Day\ 3 = Day\ Off \mid Day\ 2 = Workday)P(Day\ 2 = Workday) \\ + \\ P(Day\ 3 = Day\ Off \mid Day\ 2 = Day\ Off)P(Day\ 2 = Day\ Off) \end{pmatrix}}^{\text{Total Probability}}$$

$$= \begin{pmatrix} (0.1)(0.9) \\ + \\ (0.25)(0.1) \end{pmatrix}$$

$$= 0.115$$

e. (40 Points) What is the probability that the fourth day was a Workday (i.e. $P(Day\ 4 = Workday)$)?

$$P(Day\ 4 = Workday) = \overbrace{\begin{pmatrix} P(Day\ 4 = Workday \mid Day\ 3 = Workday)\,P(Day\ 3 = Workday) \\ + \\ P(Day\ 4 = Workday \mid Day\ 3 = Day\ Off)\,P(Day\ 3 = Day\ Off) \end{pmatrix}}^{\text{Total Probability}}$$

$$= \begin{pmatrix} (0.9)(0.885) \\ + \\ (0.75)(0.115) \end{pmatrix}$$

$$= 0.88275$$

f. (50 Points) What is the stationary distribution for a Workday (i.e. $P(Day \infty = Workday)$)?

$$P(Day \infty = Workday) \Rightarrow \underbrace{P(Day\ t = Workday)}_{x} = \underbrace{P(Day\ t-1 = Workday)}_{x}$$

$$\underbrace{P(Day\ t = Workday)}_{x} = \begin{pmatrix} P(Day\ t = Workday \mid Day\ t-1 = Workday)\underbrace{P(Day\ t-1 = Workday)}_{x} \\ + \\ P(Day\ t = Workday \mid Day\ t-1 = Day\ Off)\underbrace{P(Day\ t-1 = Day\ Off)}_{1-x} \end{pmatrix}$$

$$x = \begin{pmatrix} (0.9)(x) \\ + \\ (0.75)(1-x) \end{pmatrix}$$

$$x = 0.9x + 0.75 - 0.75x$$

$$x = 0.15x + 0.75$$

$$0.85x = 0.75$$

$$x = \frac{0.75}{0.85}$$

$$x = 0.882353$$

g. (50 Points) What is the stationary distribution for a Day Off (i.e. $P(Day\ \infty = Day\ Off)$)?

$$P(Day\ \infty = Day\ Off) \Rightarrow \underbrace{P(Day\ t = Day\ Off)}_{y} = \underbrace{P(Day\ t-1 = Day\ Off)}_{y}$$

$$\underbrace{P(Day\ t = Day\ Off)}_{y} = \begin{pmatrix} P(Day\ t = Day\ Off \mid Day\ t-1 = Workday)\underbrace{P(Day\ t-1 = Workday)}_{1-y} \\ + \\ P(Day\ t = Day\ Off \mid Day\ t-1 = Day\ Off)\underbrace{P(Day\ t-1 = Day\ Off)}_{y} \end{pmatrix}$$

$$y = \begin{pmatrix} (0.1)(1-y) \\ + \\ (0.25)(y) \end{pmatrix}$$

$$y = 0.1 - 0.1y + 0.25y$$

$$y = 0.15y + 0.1$$

$$0.85y = 0.1$$

$$y = \frac{0.1}{0.85}$$

$$y = 0.117647$$

2. Markov Chain (230 Points).

Let $W = Workday$ and $D = Day\ Off$. We observe the following sequence of Workdays and Days Off:

Sunday	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**
23: W	24: W	25: W	26: W	27: W	28: W	29: W
30: W	01: W	02: W	03: W	04: D	05: W	06: D
07: D	08: W	09: W	10: W	11: W	12: W	13: D
14: W	15: W	16: W	17: W	18: W	19: W	20: W
21: W	22: W	23: W	24: W	25: W	26: W	27: W

Note: Sunday the 23rd will be considered the first day (i.e. $Day\ 1$).

a. (10 Points) What is the probability that the first day is a workday (i.e. $P(Day\,1 = Workday)$)?

$$P(Day\,1 = Workday) = 1$$

b. (10 Points) What is the probability that the first day is a day off (i.e. $P(Day\,1 = Day\ Off)$)?

$$P(Day\,1 = Day\ Off) = 0$$

c. (40 Points) What is the probability that today will be a workday given that yesterday was a workday (i.e. $P(Day\ t = Workday \mid Day\ t-1 = Workday)$)?

$$Count\ of\ Workday\ \ followed\ by\ a\ Workday\ (\#WW) = 27$$
$$Count\ of\ Workday\ \ followed\ by\ a\ Day\ Off\ (\#WD) = 3$$

$$\begin{aligned} P(Day\ t = Workday \mid Day\ t-1 = Workday) &= \frac{\#WW}{\#WW + \#WD} \\ &= \frac{27}{27+3} \\ &= \frac{27}{30} \\ &= \boxed{0.9} \end{aligned}$$

d. (40 Points) What is the probability that today will be a day off given that yesterday was a workday (i.e. $P(Day\ t = Day\ Off \mid Day\ t-1 = Workday)$)?

$$Count\ of\ Workday\ \ followed\ by\ a\ Workday\ (\#WW) = 27$$
$$Count\ of\ Workday\ \ followed\ by\ a\ Day\ Off\ (\#WD) = 3$$

$$\begin{aligned} P(Day\ t = Day\ Off \mid Day\ t-1 = Workday) &= \frac{\#WD}{\#WW + \#WD} \\ &= \frac{3}{27+3} \\ &= \frac{3}{30} \\ &= \boxed{0.1} \end{aligned}$$

e. (40 Points) What is the probability that today will be a workday given that yesterday was a day off (i.e. $P\left(Day\ t = Workday \middle| Day\ t-1 = Day\ Off\right)$)?

$$Count\ of\ \ Day\ Off\ \ \ followed\ by\ a\ Workday\left(\#DW\right) = 3$$
$$Count\ of\ \ Day\ Off\ \ \ followed\ by\ a\ Day\ Off\ \left(\#DD\right) = 1$$

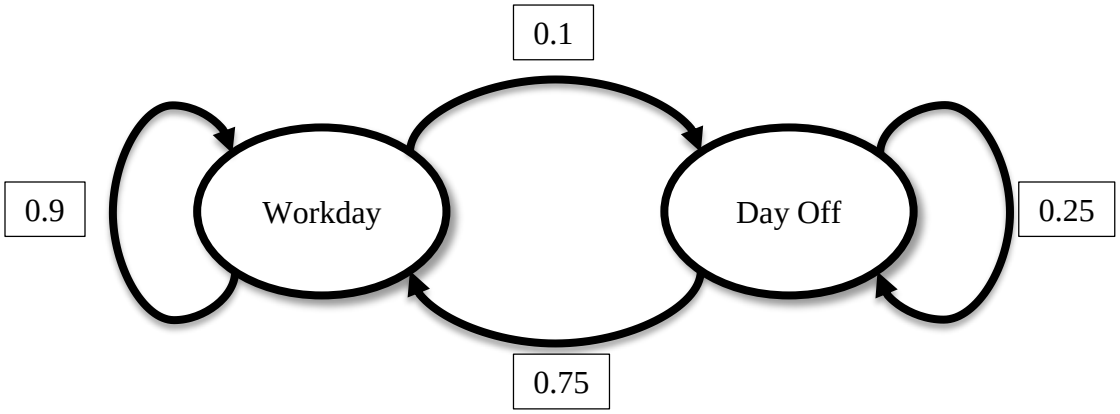
$$\begin{aligned} P\left(Day\ t = Workday \middle| Day\ t-1 = Day\ Off\right) &= \frac{\#DW}{\#DW + \#DD} \\ &= \frac{3}{3+1} \\ &= \frac{3}{4} \\ &= \boxed{0.75} \end{aligned}$$

f. (40 Points) What is the probability that today will be a day off given that yesterday was a day off (i.e. $P\left(Day\ t = Day\ Off \middle| Day\ t-1 = Day\ Off\right)$)?

$$Count\ of\ \ Day\ Off\ \ \ followed\ by\ a\ Workday\left(\#DW\right) = 3$$
$$Count\ of\ \ Day\ Off\ \ \ followed\ by\ a\ Day\ Off\ \left(\#DD\right) = 1$$

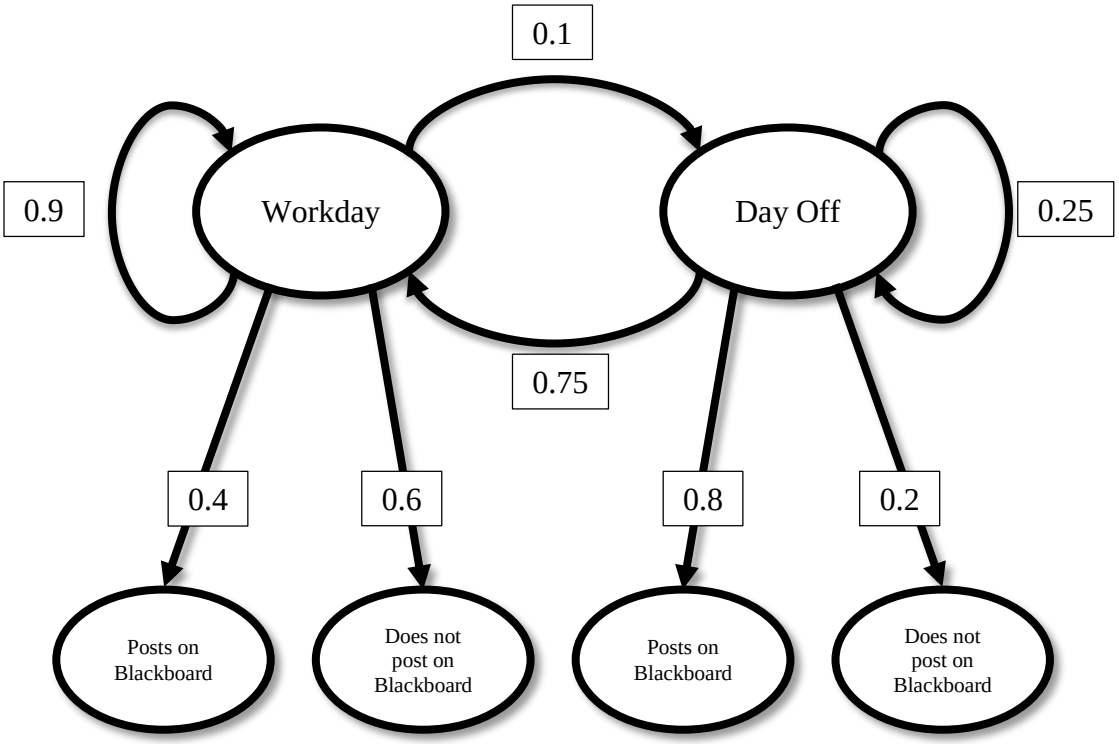
$$\begin{aligned} P\left(Day\ t = Day\ Off \middle| Day\ t-1 = Day\ Off\right) &= \frac{\#DD}{\#DW + \#DD} \\ &= \frac{1}{3+1} \\ &= \frac{1}{4} \\ &= \boxed{0.25} \end{aligned}$$

g. (50 Points) Given the data and calculations above, draw the associated Markov Chain diagram.



3. Hidden Markov Model (140 Points).

Given the following Hidden Markov Model diagram:



In this Hidden Markov Model, the observable events are whether the person of interest posts on Blackboard are does not post on Blackboard. The hidden events are that the person of interest is either working or has the day off on that day.

For the following problem, assume that:

$$P\left(Day\,1 = Workday\right) = 0.5 \text{ and } P\left(Day\,1 = Day\ Off\right) = 0.5$$

a. (10 Points) What is the probability that the person of interest posts on Blackboard on Day 2 given that they worked on Day 2 (i.e. $P\left(Day\ 2 = Posts\ on\ Blackboard \middle| Day\ 2 = Workday\right)$)?

$$P\left(Day\ 2 = Posts\ on\ Blackboard \middle| Day\ 2 = Workday\right) = \boxed{0.4}$$

b. (20 Points) What is the probability that the person of interest works on Day 2 (i.e. $P(Day\ 2 = Workday)$)?

$$P(Day\ 2 = Workday) = \overbrace{\begin{pmatrix} P(Day\ 2 = Workday \mid Day\ 1 = Workday)P(Day\ 1 = Workday) \\ + \\ P(Day\ 2 = Workday \mid Day\ 1 = Day\ Off)P(Day\ 1 = Day\ Off) \end{pmatrix}}^{\text{Total Probability}}$$
$$= \begin{pmatrix} (0.9)(0.5) \\ + \\ (0.75)(0.5) \end{pmatrix}$$
$$= \boxed{0.825}$$

c. (20 Points) What is the probability that the person of interest has the day off on Day 2 (i.e. $P(Day\ 2 = Day\ Off)$)?

$$P(Day\ 2 = Day\ Off) = \overbrace{\begin{pmatrix} P(Day\ 2 = Day\ Off \mid Day\ 1 = Workday)P(Day\ 1 = Workday) \\ + \\ P(Day\ 2 = Day\ Off \mid Day\ 1 = Day\ Off)P(Day\ 1 = Day\ Off) \end{pmatrix}}^{\text{Total Probability}}$$
$$= \begin{pmatrix} (0.1)(0.5) \\ + \\ (0.25)(0.5) \end{pmatrix}$$
$$= \boxed{0.175}$$

d. (40 Points) What is the probability that you observe the person of interest posting on Day 2 (i.e. $P(Day\ 2 = Posts\ on\ Blackboard)$)?

$$P(Day\ 2 = Posts\ on\ Blackboard) = \overbrace{\begin{pmatrix} P(Day\ 2 = Posts\ on\ Blackboard \mid Day\ 2 = Workday)P(Day\ 2 = Workday) \\ + \\ P(Day\ 2 = Posts\ on\ Blackboard \mid Day\ 2 = Day\ Off)P(Day\ 2 = Day\ Off) \end{pmatrix}}^{\text{Total Probability}}$$
$$= \begin{pmatrix} (0.4)(0.825) \\ + \\ (0.8)(0.175) \end{pmatrix}$$
$$= \boxed{0.47}$$

e. (50 Points) What is the probability that the person of interest worked given that you observed them post on Day 2 (i.e. $P\left(Day\ 2 = Workday \middle| Day\ 2 = Posts\ on\ Blackboard\right)$)?

$$P\left(Day\ 2 = Workday \middle| Day\ 2 = Posts\ on\ Blackboard\right) = \frac{P\left(Day\ 2 = Posts\ on\ Blackboard \middle| Day\ 2 = Workday\right) P\left(Day\ 2 = Workday\right)}{P\left(Day\ 2 = Posts\ on\ Blackboard\right)}$$

$$= \frac{(0.4)(0.825)}{(0.47)}$$

$$= \boxed{0.702128}$$

5. Research Project (50 Points).

Describe what the C++ code for your Research Project does.

This assignment has no programming problems.

After completing Assignment 08, create an `assignment_08_`*`lastname`*`.pdf` file for your written assignment.

Upload your `assignment_08_`*`lastname`*`.pdf` file for your written assignment to the Assignment 08 location on the BlackBoard site: https://campus.fsu.edu.