A Theory of Event Possibility with Application to Vehicle Waypoint Navigation

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Outline

- The problem being addressed
- The proposed solution: illustrative examples
- Formalization
- Possibilistic vehicle waypoint navigation

This paper abbreviates and extends results previously published as “On the possibility of an event”, *Proceedings of the 18th International Conference on Artificial Intelligence (ICAI’16)* held as part of *WorldComp’16*, July 25–28, Las Vegas, Nevada, USA, pp. 47–51.

The Problem

- Possibility Theory
  - Introduced by Zadeh in 1978.
  - Developed at length by Dubois and Prade, 1988.
  - Now enjoys a rich literature.
- Existing theory supports only subjective assignment of possibility values to events.
- Contrast probability theory which provides both subjective and objectively computable (via statistical sampling) assignment of probability values to events.
- The aim is to fill this void, i.e., to provide a computational methodology for determining the possibility degree of an event.

Proposed Solution

- The notion of possibility is context dependent.
- Events have prerequisites and constraints.
- Compute the possibility of an event as a function of the probabilities that the prerequsites are satisfied and the constraints are not.
- Implement this function using possibilistic logic.
- Thus one has a hybrid of probability and possibility theories.

Illustrative Examples

- Suppose that Jane wishes to travel to Europe next summer and her being able to go depends on her having sufficient time and money. Time and money are prerequisites. Set

$$\mathrm{Poss}(\mathrm{travel}) = \mathit{min}[\mathrm{Prob}(\mathrm{time}), \mathrm{Prob}(\mathrm{money})]$$

- Now suppose Jane has learned that a relative is ill and might need her assistance during the same time that Jane plans to travel. This would be a constraint. Here set

$$\begin{aligned}\mathrm{Poss}(\mathrm{travel}) = \mathit{min}[&\mathrm{Prob}(\mathrm{time}), \mathrm{Prob}(\mathrm{money}),\\ &\mathrm{Prob}(\neg\mathrm{assistRelative})]\end{aligned}$$

or equivalently

$$\begin{aligned}\mathrm{Poss}(\mathrm{travel}) = \mathit{min}[&\mathrm{Prob}(\mathrm{time}), \mathrm{Prob}(\mathrm{money}),\\ &1 - \mathrm{Prob}(\mathrm{assistRelative})]\end{aligned}$$

Formalization

For an event E, any proposition p can serve as a prerequisite, and any proposition c can serve as a constraint. Define the *contextual constructs* for event E by:

1. If p is a prerequisite for E, then p is a contextual construct for E.
2. If c is a constraint for E, then $(\neg c)$ is a contextual construct for E.
3. If C_1 and C_2 are contextual constructs for E, then so are $(C_1 \wedge C_2)$ and $(C_1 \vee C_2)$.

A contextual construct either of the form p where p is a prerequisite or of the form $\neg c$ where c is a constraint is an *atomic* contextual construct.

Given an event E, define the *possibility valuation* v for contextual constructs for E by:

1. If C is an atomic contextual construct for E, then $v(C) = \mathrm{Prob}(C)$.
2. If C is of the form $(C_1 \wedge C_2)$ where C_1 and C_2 are contextual constructs for E, then $v(C) = min(v(C_1), v(C_2))$.
3. If C is of the form $(C_1 \vee C_2)$ where C_1 and C_2 are contextual constructs for E, then $v(C) = max(v(C_1), v(C_2))$.

A contextual construct for an event E is *complete* if it is a full description of the relevant context for E. If C is a complete contextual construct for E, set

$$\mathrm{Poss}(E) = v(C)$$

Examples Revisited

Consider E as the first version of the event of Jane traveling to Europe. The prerequisites are p_1 = *sufficient time* and p_2 = *sufficient money* and both are required, so a complete contextual construct for E is

$$C = p_1 \wedge p_2$$

and the foregoing definitions give

$$\begin{aligned}
\mathrm{Poss}(E) &= v(C) \\
&= min(v(p_1), v(p_2)) \\
&= min(\mathrm{Prob}(p_1), \mathrm{Prob}(p_2))
\end{aligned}$$

This is the result described in the intuitive rationale.

Consider E as the more complex case of Jane's travel to Europe. Let p_1 and p_2 be as before, and let $c =$ *must assist relative*.

Then a complete contextual construct for E is

$$C = ((p_1 \wedge p_2) \wedge (\neg c))$$

Note that there can be more than one complete contextual construct depending on the manner in which these are built up from atomic constructs. For example

$$C' = ((\neg c) \wedge (p_2 \wedge p_1))$$

is also a complete contextual construct for E.

Formalization (Continued)

Given that there can be more than one complete contextual construct for the same event, the question arises whether all such constructs will evaluate to the same possibility degree.

There is no guarantee that this will be the case, since the formation of the complete contextual construct depends on how a particular user envisions the logical interrelationships between the prerequisites and constraints.

For this reason define a *context* for an event E as a complete contextual construct for E.

Then the above question becomes one of determining what conditions might be placed on contexts that would ensure that they evaluate to the same possibility degree.

The previous paper defined a notion of *strong equivalence* between contextual contexts and proved

Theorem. If two contextual constructs C and C' are strongly equivalent, then $v(C) = v(C')$.

Possibilistic Waypoint Navigation

A real-world application.

Consider the following street network, showing possible routes from waypoint A to waypoint H.

The objective is to determine the degree of possibility that a vehicle can complete the journey at a speed that is no less than the designated speed limits for the various legs.

Assume that this is a "smart city" that provides the vehicle with real-time traffic information on all the indicated legs.

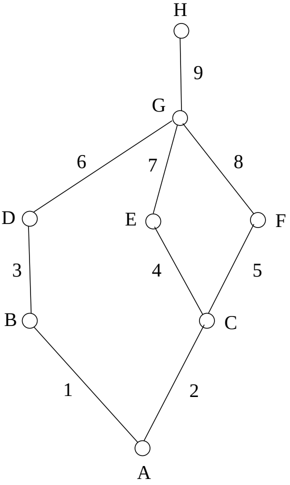

Figure: Example street network.

At point A, the vehicle must decide whether to go to B or C.

If it chooses B, the rest of the route is determined.

If it chooses C, then upon arrival at C, it must choose between E and F.

Let the preconditions for all legs be:

$$\begin{aligned} p_1 &= \text{“the vehicle is in proper working condition”} \\ p_2 &= \text{“the driver (human or robot) is competent”} \end{aligned}$$

The constraints that may cause traffic congestion on any leg are:

$$\begin{aligned} c_1 &= \text{“high traffic volume (rush hours)”} \\ c_2 &= \text{“bad weather (rain, snow, ice)”} \\ c_3 &= \text{“traffic accident”} \\ c_4 &= \text{“road construction”} \end{aligned}$$

Assume that a complete contextual construct for each leg is

$$C = p_1 \wedge p_2 \wedge \neg c_1 \wedge \neg c_2 \wedge \neg c_3 \wedge \neg c_4.$$

More exactly, to identify these items for each leg write

$$C_i = p_{1,i} \wedge p_{2,i} \wedge \neg c_{1,i} \wedge \neg c_{2,i} \wedge \neg c_{3,i} \wedge \neg c_{4,i}.$$

Then, for each i, if E_i is the event of the car traveling leg i at the designated speed limit, it follows that

$$\begin{aligned} \mathrm{Poss}(E_i) &= v(C_i) \\ &= min(\mathrm{Prob}(p_{1,i}), \mathrm{Prob}(p_{2,i}), 1 - \mathrm{Prob}(c_{1,i}), \\ &\quad 1 - \mathrm{Prob}(c_{2,i}), 1 - \mathrm{Prob}(c_{3,i}), 1 - \mathrm{Prob}(c_{4,i})). \end{aligned}$$

The indicated probabilities can vary depending on the time of day and day of week.

The value $\mathrm{Poss}(E_i)$ can be computed dynamically at any given time.

While at waypoint A, the choice is whether to proceed to B or C.

Let E_B be the event of traveling from A to H through waypoint B, and let E_C be the event of traveling through C. Then these are the composite events

$$E_B = E_1 \wedge E_3 \wedge E_6 \wedge E_9$$

$$E_C = E_2 \wedge ((E_4 \wedge E_7) \vee (E_5 \wedge E_8)) \wedge E_9$$

In accordance with the foregoing we can compute

$$\mathrm{Poss}(E_B) = min(\mathrm{Poss}(E_1), \mathrm{Poss}(E_3), \mathrm{Poss}(E_6), \mathrm{Poss}(E_9))$$

$$\begin{aligned}\mathrm{Poss}(E_C) = {} & min(\mathrm{Poss}(E_2), max(min(\mathrm{Poss}(E_4), \mathrm{Poss}(E_7)), \\ & min(\mathrm{Poss}(E_5), \mathrm{Poss}(E_8)), \mathrm{Poss}(E_9))\end{aligned}$$

and choose the path with the higher value.

If the path through waypoint C is chosen, then upon reaching that waypoint, consider $E_E = E_4 \wedge E_7 \wedge E_9$ and $E_F = E_5 \wedge E_8 \wedge E_9$, and compute

$$\mathrm{Poss}(E_E) = min(\mathrm{Poss}(E_4), \mathrm{Poss}(E_7), \mathrm{Poss}(E_9)), \text{ and}$$

$$\mathrm{Poss}(E_F) = min(\mathrm{Poss}(E_5), \mathrm{Poss}(E_8), \mathrm{Poss}(E_9))$$

and chose the path through E or F depending on which of these is higher.

In this manner one finds the path from A to H that is most possible to traverse at a speed that is at least as high as the designated speed limit.