On the Possibility of an Event

Daniel G. Schwartz

Florida State University
Tallahassee, Florida, USA

schwartz@cs.fsu.edu

Outline

- The problem to be addressed
- The proposed solution: illustrative examples
- Formalization
- Examples revisited
- Formalization continued
- Complex events
- Potential applications

The Problem

- Possibility Theory
  - Introduced by Zadeh in 1978.
  - Developed at length by Dubois and Prade, 1988.
  - Now enjoys a rich literature.
- Existing theory supports only subjective assignment of possibility values to events.
- Contrast probability theory which provides both subjective and objectively computable (via statistical sampling) assignment of probability values to events.
- The aim is to fill this void, i.e., to provide a computational methodology for determining the possibility degree of an event.

Proposed Solution

- The notion of possibility is context dependent.
- Events have prerequisites and constraints.
- Compute the possibility of an event as a function of the probabilities that the prerequsites are satisfied and the constraints are not.
- Implement this function using possibilistic logic.
- Thus one has a hybrid of probability and possibility theories.

Illustrative Examples

- Suppose that Jane wishes to travel to Europe next summer and her being able to go depends on her having sufficient time and money. Time and money are prerequisites. Set

$$\text{Poss}(\text{travel}) = \textit{min}[\text{Prob}(\text{time}), \text{Prob}(\text{money})]$$

- Now suppose Jane has learned that a relative is ill and might need her assistance during the same time that Jane plans to travel. This would be a constraint. Here set

$$\text{Poss}(\text{travel}) = \textit{min}[\text{Prob}(\text{time}), \text{Prob}(\text{money}), \text{Prob}(\neg\text{assistRelative})]$$

or equivalently

$$\text{Poss}(\text{travel}) = \textit{min}[\text{Prob}(\text{time}), \text{Prob}(\text{money}), 1 - \text{Prob}(\text{assistRelative})]$$

Examples (Continued)

- Suppose a football coach has two star quarterbacks, Bill and Bob, and believes it is possible to win as long as at least one is available. Bill and Bob are prerequisites. Set

$$\text{Poss}(\text{win}) = \textit{max}[\text{Prob}(\text{Bill}), \text{Prob}(\text{Bob})]$$

- Now suppose that Bob was injured in a previous game and may not be ready to play in the next one. This is a constraint. Set

$$\text{Poss}(\text{win}) = \textit{max}[\text{Prob}(\text{Bill}), \text{Prob}(\neg\text{BobInjured})]$$

or equivalently

$$\text{Poss}(\text{win}) = \textit{max}[\text{Prob}(\text{Bill}), 1 - \text{Prob}(\text{BobInjured})]$$

Examples (Continued)

- This rationale can be extended to more complex cases.
- Suppose that Jane has decided additionally that she does not want to travel alone and will be willing to go only if at least one of her friends Jill and John agree to accompany her. The related formula would be

$$\begin{aligned}\text{Poss}(\text{travel}) = \mathit{min}[&\text{Prob}(\text{time}), \text{Prob}(\text{money}),\\ &1 - \text{Prob}(\text{assistRelative}),\\ &\mathit{max}[\text{Prob}(\text{Jill}), \text{Prob}(\text{John})]]\end{aligned}$$

Formalization

For an event E, any proposition p can serve as a prerequisite, and any proposition c can serve as a constraint. Define the *contextual constructs* for event E by:

1. If p is a prerequisite for E, then p is a contextual construct for E.
2. If c is a constraint for E, then $(\neg c)$ is a contextual construct for E.
3. If C_1 and C_2 are contextual constructs for E, then so are $(C_1 \wedge C_2)$ and $(C_1 \vee C_2)$.

A contextual construct either of the form p where p is a prerequisite or of the form $\neg c$ where c is a constraint is an *atomic* contextual construct.

Formalization (Continued)

Given an event E, define the *possibility valuation* v for contextual constructs for E by:

1. If C is an atomic contextual construct for E, then $v(C) = \mathrm{Prob}(C)$.
2. If C is of the form $(C_1 \wedge C_2)$ where C_1 and C_2 are contextual constructs for E, then $v(C) = min(v(C_1), v(C_2))$.
3. If C is of the form $(C_1 \vee C_2)$ where C_1 and C_2 are contextual constructs for E, then $v(C) = max(v(C_1), v(C_2))$.

A contextual construct for an event E is *complete* if it is a full description of the relevant context for E. If C is a complete contextual construct for E, set

$$\mathrm{Poss}(E) = v(C)$$

Examples Revisited

Consider E as the first version of the event of Jane traveling to Europe. The prerequisites are $p_1 = \textit{sufficient time}$ and $p_2 = \textit{sufficient money}$ and both are required, so a complete contextual construct for E is

$$C = p_1 \wedge p_2$$

and the foregoing definitions give

$$\begin{aligned} \mathrm{Poss}(E) &= v(C) \\ &= \mathit{min}(v(p_1), v(p_2)) \\ &= \mathit{min}(\mathrm{Prob}(p_1), \mathrm{Prob}(p_2)) \end{aligned}$$

This is the result described in the intuitive rationale.

Examples Revisited

Consider E as the more complex case of Jane's travel to Europe. Let p_1 and p_2 be as before, let $c =$ *must assist relative*, let $p_3 =$ *Jill goes too*, and let $p_4 =$ *John goes too*.

Then a complete contextual construct for E is

$$C = ((p_1 \wedge p_2) \wedge (\neg c)) \wedge (p_3 \vee p_4)$$

Note that there can be more than one complete contextual construct depending on the manner in which these are built up from atomic constructs. For example

$$C' = (((p_4 \vee p_3) \wedge (\neg c)) \wedge (p_2 \wedge p_1))$$

is also a complete contextual construct for E.

Formalization (Continued)

Given that there can be more than one complete contextual construct for the same event, the question arises whether all such constructs will evaluate to the same possibility degree.

There is no guarantee that this will be the case, since the formation of the complete contextual construct depends on how a particular user envisions the logical interrelationships between the prerequisites and constraints.

For this reason define a *context* for an event E as a complete contextual construct for E.

Then the above question becomes one of determining what conditions might be placed on contexts that would ensure that they evaluate to the same possibility degree.

Formalization (Continued)

Contextual constructs may be regarded as propositions of the classical propositional calculus (CPC) by taking prerequisites and constraints as propositional variables.

So one might expect that contexts for the same event should be logically equivalent when thus regarded as propositions of CPC.

Unfortunately, this in itself is not sufficient to ensure that they evaluate to the same possibility degree.

It is not generally true that, if two propositions of CPC are equivalent with respect to that logic, they will also be equivalent (evaluate to the same degree) when interpreted in possibility theory. [Thanks to Vladik Kreinovich and Dana Scott].

This reduces the foregoing question to: What additional conditions on contextual constructs will ensure that they evaluate to the same possibility degree.

Formalization (Continued)

Let *conv* be the well-known algorithm that converts a proposition of CPC into conjunctive normal form (CNF).

Say that two propositions p and q of CPC are *strongly equivalent* if $conv(p)$ and $conv(q)$ can be converted into one another using only the Commutative and Associative Laws.

Say that two contextual constructs C and C', are *strongly equivalent* if they are strongly equivalent when considered as propositions of CPC.

Theorem. If two contextual constructs C and C' are strongly equivalent, then $v(C) = v(C')$.

Complex Events

We have defined a notion of possibility for a simple event whose occurrence is predicated on a set of prerequisites and/or constraints.

This gives rise to the issue of a complex event whose possibility is predicated on the possibilities of some precursor events.

More exactly, consider an event E whose possibility depends on the possibility of some Boolean combination E' of precursor events $E_1, \ldots, E_n$, where it is assumed that possibility values for $E_1, \ldots, E_n$ are known.

At issue is how to compute $\mathrm{Poss}(E)$ given $\mathrm{Poss}(E_1), \ldots, \mathrm{Poss}(E_n)$.

The foregoing theory can be extended in a natural way.

Potential Applications

While probability theory lends itself to scientific studies, possibility theory lends itself to planning.

In order to plan a course of action leading to some goal, it is of interest to know which plans are possible and, among these, which are more possible than others.

To this author's knowledge the opportunities for employing possibility theory in the area of planning have yet to be explored.

It seems reasonable that this could have use in organizational planning as well in robotics for automated mission planning and replanning.

The ideas presented in this short paper may be a step in this direction.