Horn Clauses and Prolog

Literal: a positive (not negated) or negative (negated) atomic formula. E.g., if P is atomic, both P and $\neg P$ are literals.

Horn clause: a disjunction of literals, at most one of which is positive.

Prolog statements translate into Horn clauses (and vice versa) as follows:

$$P :- P_1, P_2, \ldots, P_n$$

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \rightarrow P$$

$$\neg(P_1 \wedge \ldots \wedge P_n) \vee P \qquad A \rightarrow B \equiv \neg A \vee B$$

$$\neg P_1 \vee \ldots \vee \neg P_n \vee P \qquad \text{De Morgan's law}$$

$$:- P_1, P_2, \ldots, P_n$$

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \rightarrow F$$

$$\neg(P_1 \wedge \ldots \wedge P_n) \vee F \qquad A \rightarrow B \equiv \neg A \vee B$$

$$\neg(P_1 \wedge \ldots \wedge P_n) \qquad A \vee F \equiv A$$

$$\neg P_1 \vee \ldots \vee \neg P_n \qquad \text{De Morgan's law}$$

Also,

$$a(X, Y) :- b(X, Z), b(Y, Z)$$

can be interpeted as expressing either

$$\forall X, Y, Z(b(X, Z) \wedge b(Y, Z) \rightarrow a(X, Y))$$

or

$$\forall X, Y(\exists Z(b(X, Z) \wedge b(Y, Z)) \rightarrow a(X, Y))$$

This is based on the fact that, if x does not occur (free) in Q, then

$$\forall x(P \rightarrow Q) \text{ and } \exists x P \rightarrow Q$$

are logically equivalent. This can be verified as follows:

$$\forall x(P \rightarrow Q)$$

$$\forall x(\neg P \vee Q) \qquad A \rightarrow B \equiv \neg A \vee B$$

$$\forall x \neg P \vee Q \qquad \text{follows because } x \text{ is not free in } Q$$

$$\neg \exists x P \vee Q \qquad \forall\neg \text{ is equivalent to } \neg\exists$$

$$\exists x P \rightarrow Q \qquad A \rightarrow B \equiv \neg A \vee B$$