

3x3 Knight's Tour, p. 204-209

① $\forall X \text{ path}(X, X)$

② $\forall X, Y [\text{path}(X, Y) \leftarrow \exists Z [\text{move}(X, Z) \wedge \text{path}(Z, Y)]]$

Using $\forall X, Y (\exists Z P \rightarrow Q) \equiv \forall X, Y, Z (P \rightarrow Q)$

if Z does not occur in Q

Rewrite ② as:

②' $\forall X, Y, Z [\text{path}(X, Y) \leftarrow [\text{move}(X, Z) \wedge \text{path}(Z, Y)]]$

Using $\forall X P$ iff P , drop quantifiers, giving

① $\text{path}(X, X)$

② $\text{path}(X, Y) \leftarrow [\text{move}(X, Z) \wedge \text{path}(Z, Y)]$

Goal: $\text{path}(1, 2)$

Unify with ② by $\{1/X, 2/Y\}$, giving

$\text{path}(1, 2) \leftarrow [\text{move}(1, Z) \wedge \text{path}(Z, 2)]$

Subgoal: $\text{move}(1, Z)$

Unify with fact $\text{move}(1, 8)$ by $\{8/Z\}$,

satisfying this subgoal and giving new

Subgoal: $\text{path}(8, 2)$.

Unify with ② by $\{8/X, 2/Y\}$, giving

$\text{path}(8, 2) \leftarrow \text{move}(8, Z) \wedge \text{path}(Z, 2)$

Subgoal: $\text{move}(8, Z)$

Unify with fact $\text{move}(8, 3)$ by $\{3/Z\}$

satisfying this subgoal and giving new

Subgoal: $\text{path}(3, 2)$

Unify with ② by $\{3/X, 2/Y\}$, giving

$\text{path}(3, 2) \leftarrow \text{move}(3, Z) \wedge \text{path}(Z, 2)$

Subgoal: $\text{move}(3, Z)$

Unify with fact $\text{move}(3,4)$ by $\{4/z\}$
satisfying this subgoal and giving new

Subgoal: $\text{path}(4,2)$

Unify with $\textcircled{\text{II}}$ by $\{4/x, z/y\}$, giving
 $\text{path}(4,2) \leftarrow \text{move}(4,z) \wedge \text{path}(z,2)$

Subgoal: $\text{move}(4,z)$

Unify with fact $\text{move}(4,9)$ by $\{9/z\}$
satisfying this subgoal and giving new

Subgoal: $\text{path}(9,2)$

Unify with $\textcircled{\text{II}}$ by $\{9/x, z/y\}$, giving
 $\text{path}(9,2) \leftarrow \text{move}(9,z) \wedge \text{path}(z,2)$

Subgoal: $\text{move}(9,z)$

Unify with fact $\text{move}(9,2)$ by $\{z/z\}$
satisfying this subgoal and giving new

Subgoal: $\text{path}(z,2)$

Unify with $\textcircled{\text{I}}$ by $\{z/x\}$, satisfying this subgoal.

All unsatisfied subgoals now become satisfied in reverse order,
ultimately satisfying the initial goal $\text{path}(1,2)$.

So the answer to the query $\text{path}(1,2)$, is "yes".