

6th Edition, p. 147

Prop. If the h component of an evaluation function f is monotone, then f is monotonically nondecreasing.

Proof. Let $n_1, \dots, n_k$ be some search path in the state space.

By definition of f , for any i ,

$$f(n_i) = g(n_i) + h(n_i) \quad (1)$$

$$f(n_{i+1}) = g(n_{i+1}) + h(n_{i+1}) \quad (2)$$

From (1) we have

$$f(n_i) = g(n_i) + h(n_i) - h(n_{i+1}) + h(n_{i+1}) \quad (3)$$

Note that $g(n_{i+1}) = g(n_i) + \text{cost}(n_i, n_{i+1})$. Thus, from (2) we have

$$f(n_{i+1}) = g(n_i) + \text{cost}(n_i, n_{i+1}) + h(n_{i+1}) \quad (4)$$

Since h is monotone, we have

$$h(n_i) - h(n_{i+1}) \leq \text{cost}(n_i, n_{i+1}).$$

This gives

$$\begin{aligned} g(n_i) + (h(n_i) - h(n_{i+1})) + h(n_{i+1}) \\ \leq g(n_i) + \text{cost}(n_i, n_{i+1}) + h(n_{i+1}). \end{aligned}$$

From (3) and (4), this gives

$$f(n_i) \leq f(n_{i+1}).$$

Thus, as i increases, so does $f(n_i)$.

6th Edition, p. 148

Prop. If the h component of an evaluation function f is monotone, then the associated Algorithm A is admissible.

Proof. Let $s_1, \dots, s_g$ be a sequence of states in a search path, where s_1 is the start state and s_g is the goal. By the monotone property, we have

$$h(s_1) - h(s_2) \leq \text{cost}(s_1, s_2)$$

$$h(s_2) - h(s_3) \leq \text{cost}(s_2, s_3)$$

$\vdots$

$$h(s_{g-1}) - h(s_g) \leq \text{cost}(s_{g-1}, s_g)$$

Summing both sides gives

$$h(s_1) - h(s_g) \leq \text{cost}(s_1, s_g) \quad (1)$$

Note that $\text{cost}(s_1, s_g) = h^*(s_1)$, by definition of h^* .

Also, $h(s_g) = 0$, since h is monotone. Thus from (1) we have

$$h(s_1) \leq h^*(s_1).$$

This means that our algorithm A is actually algorithm A^* , in which case A is admissible by the result cited on p. 146.

6th ed, p. 158

$$T = B + B^2 + \dots + B^L$$

$$= B(1 + B + \dots + B^{L-1})$$

$$= B(1 + B + \dots + B^{L-1} + B^L - B^L)$$

$$= B(1 + T - B^L)$$

$$= B + BT - B^{L+1}$$

$$T - BT = B - B^{L+1}$$

$$T(1 - B) = B - B^{L+1}$$

$$T = \frac{B - B^{L+1}}{1 - B}$$

$$= \frac{B - B^{L+1}}{1 - B} \cdot \frac{-1}{-1}$$

$$= \frac{B^{L+1} - B}{B - 1}$$

$$= \frac{B(B^L - 1)}{B - 1}$$