COT5405: Fall 2006

Lecture 7

LP Duality and Slackness

Note: Some of the statements here are informal. Please read the handout from **AA** for the formal statements and proofs.

LP Duality Theorem: If x^* is an optimum solution for the primal and y^* is an optimum solution for the dual, then

$$\Sigma_j c_j x_j = \Sigma_i b_i y_i.$$

Weak Duality Theorem: If x is a feasible solution for the primal and y is a feasible solution for the dual, then

$$\Sigma_j c_j x_j \geq \Sigma_i b_i y_i.$$

Proof:

$\Sigma_j c_j x_j \geq \Sigma_j (\Sigma_i a_{ij} y_i)\, x_j$, because x_js are non-negative, and y is feasible for the dual. *(1)*

Similarly,

$\Sigma_i b_i y_i \leq \Sigma_i (\Sigma_j a_{ij} x_j)\, y_i = \Sigma_j (\Sigma_i a_{ij} y_i)\, x_j.$ *(2)*

From (1) and (2), $\Sigma_j c_j x_j \geq \Sigma_i b_i y_i$.

Complementary slackness conditions: Let x be a feasible solution for the primal and let y be a feasible solution for the dual. Then x and y are both optimum iff both the following conditions are satisfied.

1. For each j, $x_j = 0$ or $\Sigma_i a_{ij} y_i = c_j$, and
2. For each i, $y_i = 0$ or $\Sigma_j a_{ij} x_j = b_i$.

- *Can you show that the above two conditions imply that x and y are optimum solutions, if they are both feasible, using the weak duality theorem?*
- *Can you show that if x and y are optimum solutions, then the above two conditions hold, using the LP duality theorem?*