COT5405: Fall 2006

Lecture 6

A linear programming example

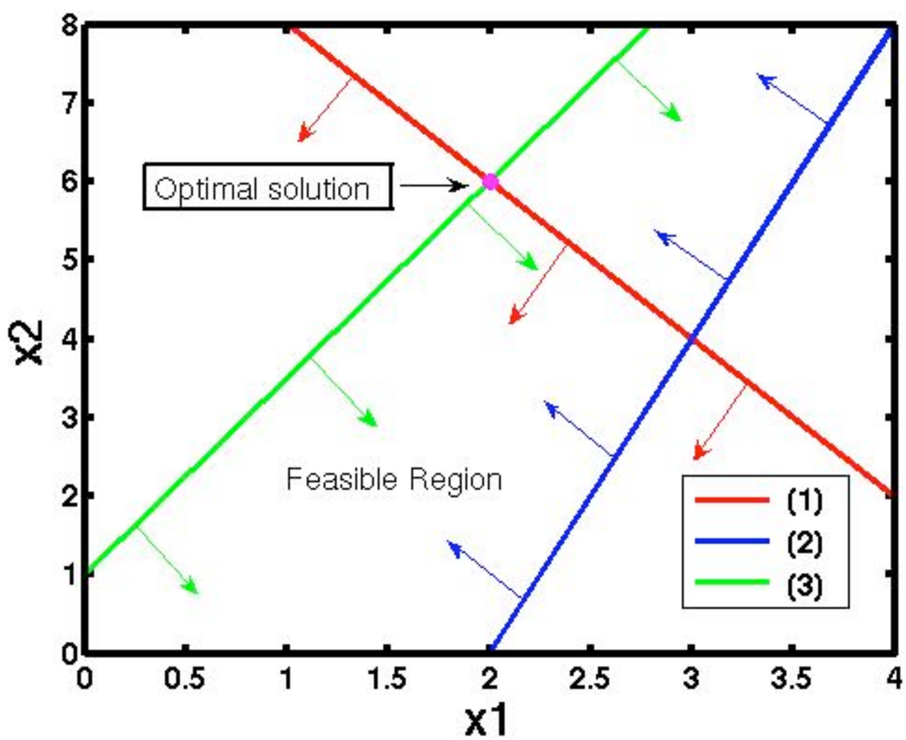
Objective function:

Maximize $x_1 + x_2$

Constraints: (Subject to)

$2\,x_1 + \;\; x_2 \leq \; 10$ (1)
$4\,x_1 - \;\; x_2 \leq \;\; 8$ (2)
$5\,x_1 - 2\,x_2 \geq -2$ (3)

$x_1, x_2 \geq 0$ **Non-negativity constraint**

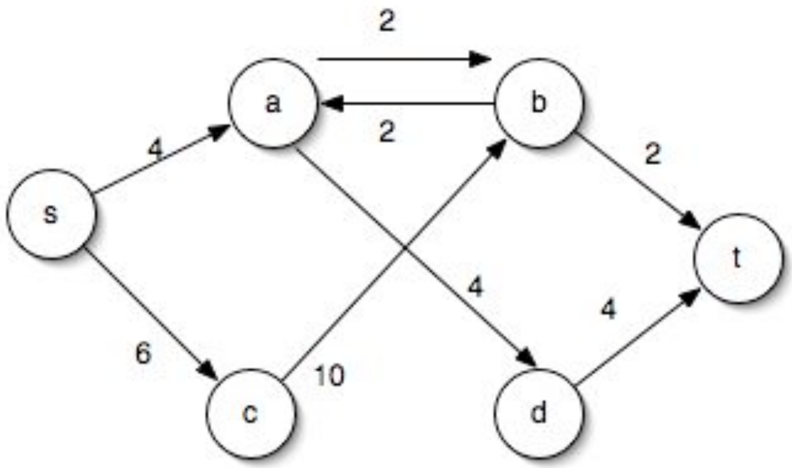


- *Optimal solution* is (2,6) with optimal *objective value* 8.
- (0,0) is a *feasible solution* with objective value 0.

Integer linear program: x_i should be integers.

Formulating Max-Flow as a Linear Program

Max-Flow example:



We are given a directed graph *(V, E)* with a non-negative capacity *c* for each edge *(u, v)* in *E*. The numbers on the *edges* give the capacities, in the above figure. We can take the capacity for edges *not in E* as *0*. The vertex **s** above is called the *source*, and the vertex **t** is called the *sink*. We want to maximize the flow from the source to the sink by assigning flow values *f* to each edge satisfying the following constraints:

- **Capacity constraint:** For each u, v in V, $f(u, v) \leq c(u, v)$
- **Skew symmetry:** For all u, v in V, $f(u, v) = -f(v, u)$
- **Flow Conservation:** For all u in $V - \{s, t\}$, $\Sigma_{v \text{ in } V} f(u, v) = 0$

We wish to maximize $\Sigma_{v \text{ in } V} f(s, v)$. For more details, see section 26.1 of CLR.

Formulation of Max-Flow as a linear program:

Let us introduce variables f_{uv} corresponding to the flows f(u, v) for each u, v in V. Let c_{uv} = c(u, v).

Maximize: $\Sigma_{v \text{ in } V} f_{sv}$

Subject to:

$f_{uv} \leq c_{uv}$ for all u, v in V
$f_{uv} = -f_{vu}$ for all u, v in V
$\Sigma_{v \text{ in } V} f_{vu} = 0$ for all u in $V - \{s, t\}$