COT5405: Fall 2006

Lecture 9

Approximation guarantee for the primal-dual algorithm

Note: Some of the statements here are informal. Please read the handout from **AA** for the formal statements and proofs.

Primal	**Dual**
Min: $\Sigma_j c_j x_j$ Subject to: $\Sigma_j a_{ij} x_j \geq b_i$, i = 1, ..., m $x_j \geq 0$, j = 1, ..., n	Max: $\Sigma_i b_i y_i$ Subject to: $\Sigma_i a_{ij} y_i \leq c_j$, j = 1, ..., n $y_i \geq 0$, j = 1, ..., n
Complementary slackness	
$x_j = 0$ or $\Sigma_i a_{ij} y_i = c_j$	$y_i = 0$ or $\Sigma_j a_{ij} x_j = b_i$
Relaxed slackness	
$x_j = 0$ or $c_j/\alpha \leq \Sigma_i a_{ij} y_i \leq c_j$, $\alpha \geq 1$ *(1)*	$y_i = 0$ or $b_i \leq \Sigma_j a_{ij} x_j \leq \beta b_i$, $\beta \geq 1$ *(2)*

Proposition 15.1: If x is feasible for the primal, y is feasible for the dual, and they satisfy the relaxed slackness conditions (1) and (2), then $\Sigma_j c_j x_j \leq \alpha\beta\, \Sigma_i b_i y_i \leq \alpha\beta\, OPT$.

Proof:

$\Sigma_j c_j x_j \leq \alpha\, \Sigma_j (\Sigma_i a_{ij} y_i)\, x_j$ (from (1))
$= \alpha\, \Sigma_i \Sigma_j a_{ij} y_i\, x_j = \alpha\, \Sigma_i y_i\, \Sigma_j a_{ij}\, x_j$
$\leq \alpha\beta\, \Sigma_i y_i b_i$ (from (2)) $\leq \alpha\beta\, OPT$.

Note: Even though we can take $c_j/\alpha \leq \Sigma_i a_{ij} y_i$ only when $x_j > 0$, $x_j\, c_j/\alpha \leq x_j\, \Sigma_i a_{ij} y_i$ when $x_j \geq 0$. Similarly, $y_i\, \Sigma_j a_{ij} x_j \leq y_i\, \beta b_i$ when $y_i \geq 0$.

Approximation factor for Set Cover Primal-Dual approximation algorithm: Its relaxed slackness conditions have $\alpha = 1$ and $\beta = f$ in proposition 15.1, yielding a factor f approximation.