COT5405: Fall 2006

Lecture 5

Proof for Lemma 8.3

Lemma 2.3: *The set S' , output by the algorithm, satisfies:*

$profit(S') \geq (1-\varepsilon)\ OPT$

Proof: Note the following.

$$profit'(a_i) + 1 \geq profit(a_i)/k => k\ profit'(a_i) + k \geq profit(a_i), \tag{1}$$

because the floor function is used in computing *profit'*.

$$profit(a_i) \geq k\ profit'(a_i), \tag{2}$$

for the same reason as above.

Let O be the optimal solution for the original instance. Then,

$$profit(O) - k\ profit'(O) \leq nk \text{ (from (1))} \tag{3}$$
and
$profit(S') \geq k\ profit'(S') \text{ (from (2))} \geq k\ profit'(O) \text{ (from optimality of S' for } profit')$
$$\geq profit(O) - nk \text{ (from (3))} = OPT - \varepsilon\mathcal{P}n/n = OPT - \varepsilon\mathcal{P}. \tag{4}$$

Observing that $OPT \geq \mathcal{P}$, we get the following from (4).

$profit(S') \geq OPT - \varepsilon\ OPT = (1-\varepsilon)\ OPT.$