COP 4531 – Assignment #3

Fall 2005

(maximum score = 40)

1. Show that for any real constants a and b, where $b > 0$, $(n + a)^b = \Theta(n^b)$. (8 points)

2. Show that if $f(n)$ and $g(n)$ are monotonically increasing functions, then so are the functions $f(n) + g(n)$ and $f(g(n))$, and if $f(n)$ and $g(n)$ are in addition nonnegative, then $f(n) * g(n)$ is monotonically increasing. (6 points)

3. Use the master method to give tight asymptotic bounds for the following recurrences. (6 points)

    a. $T(n) = 4T(n/2) + n$.
    b. $T(n) = 4T(n/2) + n^2$.
    c. $T(n) = 4T(n/2) + n^3$.

4. The recurrence $T(n) = 7T(n/2) + n^2$ describes the running time of an algorithm A. Algorithm B has a running time of $T(n) = aT(n/4) + n^2$. What is the largest integer value for a such that B is asymptotically faster than A? (8 points)

5. Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n < 3$. Make your bounds as tight as possible, and justify your answers. (12 points)

    a. $T(n) = 2T(n/2) + n^3$.
    b. $T(n) = T(9n/10) + n$.
    c. $T(n) = 16T(n/4) + n^2$.
    d. $T(n) = 7T(n/3) + n^2$.
    e. $T(n) = 7T(n/2) + n^2$.
    f. $T(n) = 2T(n/4) + n^{1/2}$.