# Functional Programming

- Why functional programming?
- Historical origins of functional programming
- Functional programming today
- Concepts of functional programming
- A crash course on programming in Scheme

Note: this set of notes covers Chapter 11 Sections 11.1 to 11.2. You are not required to study Sections 11.2.2, 11.2.4, and 11.2.5.

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## Why Functional Programming?

- Imperative programming languages are more widely used
  - Integrated software development environments for procedural and object oriented programming languages are "industrial strength"
- However, (commercial) applications exist for functional programming:
  - Symbolic data manipulation
  - Natural language processing
  - Artificial intelligence
  - Automatic theorem proving and computer algebra

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## Why Functional Programming in This Course?

- A functional language will be used to illustrate a diversity of programming language concepts
- Functional programming languages are
  - Compiled and/or interpreted (Section 1.4)
  - Have simple syntax (Chapter 2)
  - Use garbage collection (Section 3.2.3) for memory management
  - Are statically scoped or dynamically scoped (Section 3.3)
  - Use higher-order functions and subroutine closures (Section 3.4.1)
  - Use first-class function values (Section 3.4.2)
  - Depend heavily on polymorphism (Section 3.5)
  - Employ recursion (Section 6.6) for repetitive execution
  - Programs have no side effects and all expressions are referentially transparent (Sections 6.1.2 and 6.3)

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## Origin of Functional Programming

- Church’s thesis:
  - All models of computation are equally powerful and can compute any function
- Turing’s model of computation: Turing machine
  - Reading/writing of values on an infinite tape by a finite state machine
- Church’s model of computation: lambda calculus
  - This inspired functional programming as a concrete implementation of lambda calculus
- Computability theory
  - A program can be viewed as a constructive proof that some mathematical object with a desired property exists
  - A function is a mapping from inputs to output objects and computes output objects from appropriate inputs
  - For example, the proposition that every pair of nonnegative integers (the inputs) has a greatest common divisor (the output object) has a constructive proof implemented by Euclid’s algorithm written as a "function"

\[
\text{gcd}(a, b) = \begin{cases} 
  a & \text{if } a = b \\
  \text{gcd}(a-b, b) & \text{if } a > b \\
  \text{gcd}(a, b-a) & \text{if } b > a
\end{cases}
\]
### Functional Programming Today

- Attractive model of computation
  - Absence of *side effects* makes expressions referentially transparent: the value of an expression depends solely on the function return values in it and not on evaluation order and/or values of global variables
  - A function can always be counted on to return the same results with the same input parameters
  - Dangling and/or uninitialized pointer references do not occur
- Significant improvements in theory and practice of functional programming have been made in recent years
  - Easier to write functional programs by using their imperative language features which are automatically translated to functional constructs (e.g. loops by recursion)
  - Improved efficiency
- Remaining obstacles to functional programming:
  - *Social*: not many libraries, not very portable, and no integrated development environments for functional languages

### Concepts of Functional Programming

- *Functional programming* defines the outputs of a program as mathematical function of the inputs with no notion of internal state (no side effects)
  - Example: *pure* functional programming languages: Miranda, Haskell, and Sisal
- Non-pure functional programming languages include imperative features with side effects that affect global state (e.g. through destructive assignments to global variables)
  - Example: Lisp, Scheme, and ML
- Useful features are found in functional languages that are often missing in *imperative* languages:
  - *First-class function values*: the ability of functions to return newly constructed functions
  - *Higher-order functions*: functions that take other functions as input parameters or return functions
  - Polymorphism: the ability to write functions that operate on more than one type of data
  - Aggregate constructs for constructing structured objects: the ability to specify a structured object in-line, e.g. a complete list or record value
  - Garbage collection

### Lisp

- *Lisp* (LIst Processing language) was the original functional language
- Lisp and dialects are still the most widely used
- Simple and elegant design of Lisp:
  - *Homogeneity of programs and data*: a Lisp program is a list and can be manipulated in Lisp as a list
  - *Self-definition*: a Lisp interpreter can be written in Lisp
  - *Interactive*: interaction with user through "read-eval-print" loop

### A Crash Course on Scheme

- Scheme is a popular Lisp dialect
- Lisp and Scheme adopt *Cambridge Polish notation* for expressions:
  - An expression is an *atom*, e.g. a number, string, or identifier name
  - An expression is a *list* whose first element is the function name (or operator) followed by the arguments which are expressions:
    - `(function arg1 arg2 arg3 ...)`
  - The "Read-eval-print" loop provides user interaction: an expression is read, evaluated by evaluating the arguments first and then the function/operator is called after which the result is printed
    - Input: 9
    - Output: 9
    - Input: (+ 3 4)
    - Output: 7
    - Input: (+ (* 2 3) 1)
    - Output: 7
- User can load a program from a file with the `load` function
  - `(load "my_scheme_program")`
  - The file name should use the `.scm` extension

Note: You can run the Scheme interpreter and try the examples in these notes by executing the `scheme` command. To exit Scheme, type `(exit)`. You can download an example Scheme program "Eliza".

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### Garbage Collection

Garbage collection is an automatic process that removes objects from memory that are no longer needed. It helps to free up memory for other uses and prevent memory leaks.

- **Definition**: The garbage collector identifies and removes objects that are no longer accessible to the program.
- **Benefits**:
  - Prevents memory leaks by automatically freeing up memory.
  - Improves memory efficiency by reusing memory for new objects.
- **Types**:
  - **Runtime**: Garbage collection that occurs during program execution.
  - **Preemptive**: Garbage collection that may be initiated by the operating system.

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### Aggregate Constructs for Constructing Structured Objects

Aggregate constructs enable the creation of structured objects in programming languages.

- **Definition**: The ability to specify a structured object in-line, e.g., a complete list or record value.
- **Examples**:
  - In Scheme, aggregate constructs are used to define data structures such as lists or records.
  - Scheme supports the definition of structured objects using `define` or `let`.

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### Garbage Collection Example

- **Algorithm**: Garbage collection algorithms are designed to efficiently identify and remove garbage.
- **Common Algorithms**:
  - *Mark and Sweep*:
    - Mark phase: marks all objects reachable from the roots.
    - Sweep phase: frees all unmarked objects.
  - *Collecting Garbage*:
    - Collects garbage in a set of objects that are not on the heap.

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### Lisp Example

- **Lisp**: A functional programming language that uses a non-strict evaluation strategy.
- **Key Features**:
  - **Higher-order Functions**: Functions that can take other functions as arguments or return functions as results.
  - **Cambridge Polish Notation**: Expressions are written in postfix notation, with function symbols preceding their arguments.

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### Scheme Example

- **Scheme**: A dialect of Lisp that emphasizes simplicity and elegance.
- **Interactive**: Scheme provides an interactive environment where users can load and execute programs.
- **Examples**:
  - `(read)` reads an expression from the input.
  - `(eval expr)` evaluates an expression.
  - `(print expr)` prints the value of an expression.

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### Garbage Collection Summary

Garbage collection is a critical aspect of memory management in programming languages.

- **Benefits**: Prevents memory leaks, improves memory efficiency, and simplifies program development.
- **Types**: Runtime vs. Preemptive Garbage Collection.
- **Algorithms**: Mark and Sweep, Collecting Garbage.

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### Lisp Summary

Lisp is a functional programming language with a rich history and a significant influence on computer science.

- **Features**:
  - **Higher-order Functions**: Functions that can manipulate other functions.
  - **Cambridge Polish Notation**: Expressions written in a non-strict evaluation order.
  - **Interactive Environment**: Users can interact with the language through a read-eval-print loop.

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### Scheme Summary

Scheme is a popular dialect of Lisp that emphasizes simplicity and ease of use.

- **Features**:
  - **Interactive**: Users can interact with the language through a command-line interface.
  - **Load Function**: Allows loading programs from files.
  - **Extension**: `.scm` extension for file names.

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### Garbage Collection Application

Garbage collection is essential for managing memory in modern programming languages.

- **Benefits**: Prevents memory leaks, improves memory efficiency, and simplifies memory management.
- **Challenges**: Determining which objects are still accessible and should be kept in memory.
- **Techniques**: 1) Mark and Sweep, 2) Collecting Garbage.

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### Lisp Application

Lisp is widely used in artificial intelligence, expert systems, and other domains that require symbolic computation.

- **Applications**:
  - **AI**:用于符号计算和自然语言处理。
  - **Symbolic Computation**: Simulating human thinking processes.
  - **Expert Systems**: Solving complex problems with heuristics.

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### Scheme Application

Scheme is used in various domains, including education and research.

- **Applications**:
  - **Education**: Teaching programming concepts.
  - **Research**: Supporting research in areas like functional programming.
  - **Interactive Environments**: Providing a platform for experimentation and learning.

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### Garbage Collection Further Reading

- Garbage collection: a fundamental concept in computer science.
- Understanding how garbage collection works can improve your programming skills.
- Reading about different garbage collection algorithms can provide insights into memory management strategies.

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### Lisp Further Reading

- Lisp's elegant design and expressive nature make it a preferred choice for developers.
- Learning Lisp can improve your understanding of functional programming concepts.
- Exploring other dialects of Lisp can broaden your perspective on functional programming languages.

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### Scheme Further Reading

- Scheme's simplicity and elegance make it an accessible language for learning.
- Understanding Scheme can improve your ability to understand and use other functional programming languages.
- Exploring the different dialects of Scheme can provide insights into the evolution of functional programming languages.
Scheme Data Structures

- The only data structures in Lisp and Scheme are **atoms** and **lists**
- **Atoms** are:
  - Numbers, e.g. 7
  - Strings, e.g. "abc"
  - Identifier names (variables), e.g. x
  - Boolean values true #t and false #f
  - Symbols which are quoted identifiers which will not be evaluated, e.g. 'y
    - Input: a
    - Output: Error: unbound variable a
    - Input: 'a
    - Output: a
- **Lists**:
  - To distinguish list data structures from expressions that are written as lists, a quote (') is used to quote the list:
    - Input: '(elt1 elt2 elt3 ...)
    - Output: (elt1 elt2 elt3 ...)
  - Input: '(3 4 5)
    - Output: (3 4 5)
  - Input: '(a 6 (x y) "a")
    - Output: (a 6 (x y) "a")
  - Input: '(a (+ 3 4))
    - Output: (a (+ 3 4))
  - Input: '()
    - Output: ()
- Note: the empty list () is also identical to false #f in Scheme

Primitive List Operations

- **car** returns the head (first element) of a list
  - Input: (car '(2 3 4))
  - Output: 2
- **cdr** (pronounced "coulter") returns the tail of a list (list without the head)
  - Input: (cdr '(2 3 4))
  - Output: (3 4)
- **cons** joins an element and a list to construct a new list
  - Input: (cons 2 '(3 4))
  - Output: (2 3 4)
- **Examples**:
  - Input: (car '())
    - Output: 2
  - Input: (car '(1))
    - Output: Error
  - Input: (cdr '(2 3))
    - Output: (3)
  - Input: (cdr (cdr '(2 3 4)))
    - Output: (4)
  - Input: (cdr '())
    - Output: ()
  - Input: (cons 2 '())
    - Output: (2)

Type Checking

- The type of an expression is determined only at run-time
- Functions need to check the types of their arguments explicitly
- Type predicate functions:
  - (boolean? x) ; is x a Boolean?
  - (char? x) ; is x a character?
  - (string? x) ; is x a string?
  - (symbol? x) ; is x a symbol?
  - (number? x) ; is x a number?
  - (list? x) ; is x a list?
  - (pair? x) ; is x a non-empty list?
  - (null? x) ; is x an empty list?

If-Then-Else

- **Special forms** resemble functions but have special evaluation rules
- A **conditional expression** in Scheme is written using the **if** special form:
  - (if condition thenexpr elseexpr)
  - Input: (if #t 1 2)
    - Output: 1
  - Input: (if #f 1 "a")
    - Output: "a"
  - Input: (if (string? "s") (+ 1 2) 4)
    - Output: 3
  - Input: (if (> 1 2) "yes" "no")
    - Output: "no"
- A more general if-then-else can be written using the **cond** special form:
  - (cond (condition valuepair) ...)
  - Input: (cond ((< 1 2) 1) ((>= 1 2) 2))
    - Output: 1
  - Input: (cond ((< 2 1) 1) ((= 2 1) 2) (else 3))
    - Output: 3
Testing
- `eq?` tests whether its two arguments refer to the same object in memory
  - Input: `(eq? 'a 'a)`
  - Output: `#t`
  - Input: `(eq? '(a b) '(a b))`
  - Output: `()` (false: the lists are not stored at the same location in memory!)
- `equal?` tests whether its arguments have the same structure
  - Input: `(equal? 'a 'a)`
  - Output: `#t`
  - Input: `(equal? '(a b) '(a b))`
  - Output: `#t`
- To test numerical values, use `=`, `<>`, `>`, `<`, `>=`, `<=`, `even?`, `odd?`, `zero?`
  - Input: `(member 'y "s" x 3 y z)`
  - Output: `(y z)`
  - Input: `(member 'y '(x (3 y) z))`
  - Output: `()`

Lambda Abstraction
- A Scheme lambda abstraction is a nameless function specified with the `lambda` special form:
  
  ```scheme
  (lambda formalparameters functionbody)
  ```
  
  where the `formal parameters` are the function inputs and the `function body` is an expression that is the resulting value of the function
- Examples:
  - `(lambda (x) (* x x)) ; is a squaring function: x*x`
  - `(lambda (a b) (sqrt (+ (* a a) (* b b)))) ; is a function:
    
    $$(a b) \mapsto \sqrt{a^2 + b^2}$$

Lambda Application
- A lambda abstraction is applied by assigning the evaluated actual parameter(s) to the formal parameters and returning the evaluated function body
- The form of a function call in an expression is:
  
  ```scheme
  (function arg1 arg2 arg3 ...)
  ```
  
  where `function` can be a lambda abstraction
- Example:
  - Input: `((lambda (x) (* x x)) 3)`
  - Output: `9`
  - That is, $x=3$ in $(* x x)$ which evaluates to 9

Defining Global Functions in Scheme
- A function is globally defined using the `define` special form:
  
  ```scheme
  (define name function)
  ```
  
  - For example:
    - `(define sqr
      (lambda (x) (* x x)))`
    - defines function `sqr`
      - Input: `(sqr 3)`
      - Output: `9`
      - Input: `(sqr (sqr 3))`
      - Output: `81`
    - `(define hypot
      (lambda (a b)
        (sqrt (+ (* a a) (* b b))))`
      - defines function `hypot`
      - Input: `(hypot 3 4)`
      - Output: `5`


**Bindings**

- An expression can have local name-value bindings defined with the `let` special form

\[
\text{let listofnameandvaluepairs expression}
\]

where `name and value pairs` is a list of pairs `(namevalue)` and expression is returned in which each name is replaced with its value in the list.

- Input:

\[
\text{let ((a 3) (b 4)) (hypot a b)}
\]

- Output: 5

- A name can be bound to a function in `let`

- Input:

\[
\text{let ((sqr (lambda (x) (* x x))) (y 3)) (sqr y)}
\]

- Output: 9

**Recursive Bindings**

- An expression can have local recursive function bindings defined with the `letrec` special form

\[
\text{letrec listofnameandvaluepairs expression}
\]

where `name and value pairs` is a list of pairs `(namevalue)` and expression is returned where each name is replaced with its value

- Input:

\[
\text{letrec ((fact (lambda (n) (if (= n 1) 1 (* n (fact (- n 1))))) (fact 5)) (fact 5))}
\]

- Output: 120

- This allows the local factorial function `fact` to refer to itself

**I/O and Sequencing**

- `display` prints a value

- Input: `(display "Hello World!")`

- Output: "Hello World!"

- Input: `(display (+ 2 3))`

- Output: 5

- `newline` advances to a new line

- Input: `(newline)`

- `read` returns a value from standard input

- `begin` sequences a series of expressions (its value is the value of the last expression)

- Example:

\[
\text{begin (display "Hello World!") (newline)}
\]

- Example:

\[
\text{let ((x 1) (y (read)) (plus +)) (begin (display (plus x y)) (newline))}
\]

**Loops**

- `do` takes a list of name-init-update triples, a termination test with final value, and a loop body

\[
\text{do listoftriples condition body}
\]

- Example:

\[
\text{do ((i 0 (+ i 1))) ((>= i 10) *done*) (display i) (newline) (newline)}
\]

- Since everything is an expression in Scheme, a loop must return a value which in this case is the string "*done*"
Higher-Order Functions

- A function is called a **higher-order function** (also called a **functional form**) if it takes a function as an argument or returns a newly constructed function as a result.
- Scheme has several built-in higher-order functions, for example:
  - `apply` takes a function and a list and applies the function with the elements of the list as arguments.
    - Input: `(apply ' + '(3 4))`
    - Output: `7`
  - `map` takes a function and a list and returns a list after applying the function to each element of the list.
    - Input: `(map odd? '(1 2 3 4))`
    - Output: `(#t #t #f)`
- Here is a function that applies a function to an argument twice:
  - `(define twice (lambda (f n) (f (f n))))`
  - Input: `(twice sqrt 81)`
  - Output: `3`

Non-Pure Constructs: Assignments

- Assignments are considered bad in functional programming because they can change the global state of the program and possibly influence function outcomes.
  - `(set! a 0)`
    - `... (set! a (+ a 1)) ; increment a by 1 ...`
  - `(set-car! (cons 1 2))`
  - `(set-cdr! (cons 1 2))`

Scheme Examples

- Recursive factorial function:
  - `(define fact (lambda (n)
      (if (zero? n) 1 (* n (fact (- n 1))))))`
- Iterative factorial function:
  - `(define iterfact (lambda (n)
      (do ((i 1 (+ i 1))
          (f 1 (* f i))
          (> i n) f)
       ;; note: loop body is omitted))`

Example Recursive Functions on Lists

- Sum the elements of a list:
  - `(define sum (lambda (lst)
      (if (null? lst) 0
          (+ (car lst) (sum (cdr lst)))))`
- Check if element is in list:
  - `(define in? (lambda (elt lst)
      (cond ((null? lst) #f) ; if list is empty, return false
              ((= elt (car lst)) #t) ; if element is the head, return true
              (else (in? elt (cdr lst))) ; keep searching rest of list))`
- Fill a list:
  - `(define fill (lambda (num elt)
      (cond ((= num 0) '())
              (else (cons elt (fill (- num 1) elt)))))`

Examples of List Functions
Examples of Higher-Order Functions

- Reduce a list by applying a binary operator to all elements (i.e. \( elt1 + elt2 + elt3 + \ldots \)):
  ```scheme```
  ```scheme```(define reduce
  (lambda (op lst)
    (if (null? (cdr lst))
      (car lst)
      (op (car lst) (reduce op (cdr lst))))
  )
  ```scheme```
  ```scheme``` Input: (reduce + '(1 2 3))
  ```scheme```
  ```scheme``` Output: 6

- Filter elements of a list for which a condition (a predicate function) returns true:
  ```scheme```
  ```scheme```(define filter
  (lambda (op lst)
    (cond
      ((null? lst) '())
      ((op (car lst)) (cons (car lst) (filter op (cdr lst))))
      (else (filter op (cdr lst))))
  )
  ```scheme```
  ```scheme``` Input: (filter odd? '(1 2 3 4 5))
  ```scheme```
  ```scheme``` Output: (1 3 5)

- Examples of Higher-Order Functions

  - Fill a list with numbers:
    ```scheme```
    ```scheme```(define between
    (lambda (start end)
      (if (> start end)
        '()
        (cons start (between (+ start 1) end)))
    )
    ```scheme```
    ```scheme``` Input: (between 1 10)
    ```scheme```
    ```scheme``` Output: (1 2 3 4 5 6 7 8 9 10)

  - Zip two lists together:
    ```scheme```
    ```scheme```(define zip
    (lambda (lst1 lst2)
      (cond
        ((null? lst1) '())
        ((null? lst2) '())
        (else (cons (list (car lst1) (car lst2)) (zip (cdr lst1) (cdr lst2))))
      )
    )
    ```scheme```
    ```scheme``` Input: (zip '(1 2 3) '(a b c))
    ```scheme```
    ```scheme``` Output: ((1 a) (2 b) (3 c))

  - Take a certain number of elements from a list:
    ```scheme```
    ```scheme```(define take
    (lambda (num lis)
      (cond
        ((= num 0) '())
        (else (cons (car lis) (take (- num 1) (cdr lis))))
      )
    )
    ```scheme```
    ```scheme``` Input: (take 3 '(a b c d e f))
    ```scheme```
    ```scheme``` Output: (a b c)

- Reduce a list by applying a binary operator to all elements (i.e. \( elt1 + elt2 + elt3 + \ldots \)):
  ```scheme```
  ```scheme```(define reduce
  (lambda (op lst)
    (if (null? (cdr lst))
      (car lst)
      (op (car lst) (reduce op (cdr lst))))
  )
  ```scheme```
  ```scheme``` Input: (reduce + '(1 2 3))
  ```scheme```
  ```scheme``` Output: 6

- Filter elements of a list for which a condition (a predicate function) returns true:
  ```scheme```
  ```scheme```(define filter
  (lambda (op lst)
    (cond
      ((null? lst) '())
      ((op (car lst)) (cons (car lst) (filter op (cdr lst))))
      (else (filter op (cdr lst))))
  )
  ```scheme```
  ```scheme``` Input: (filter odd? '(1 2 3 4 5))
  ```scheme```
  ```scheme``` Output: (1 3 5)