This homework will be collected in class on Thursday 1st November. On Wednesday 7th November, solutions will be reviewed and handed out.

1. We have seen that if language $L$ is recursive then so is $\overline{L}$. This problem has to do with the closure properties of r.e. languages and recursive languages.
   Tell whether (a) r.e. languages, and (b) recursive languages, are closed under the following operations:
   - union,
   - intersection,
   - concatenation,
   You may give informal but clear constructions to show closure.

2. Informally define multitape TMs that enumerate the following sets of integers (in the sense that starting with blank tapes they print out on their tapes $10^i10^i1\ldots$, to represent the set $i_1,i_2,\ldots$).
   - The set of perfect squares: $\{1,4,9,\ldots\}$.
   - The set of all primes: $\{2,3,5,\ldots\}$.

3. The halting problem: given the pair $(M,w)$, where $M$ is a TM and $w \in \Sigma^*$ is a string, does $M$ halt on input $w$?
   Let $L_h$ be the language of the halting problem. Show that $L_h$ is r.e. but not recursive.
   (Hint: The universal language $L_v$ is r.e. but not recursive. Mimic the proof.)

4. Show that the following properties of r.e. sets are not decidable. Justify your answers.
   (a) $L = \emptyset$.
   (b) $L$ is recursive.
   (c) $\overline{L}$ is finite.
   Hint: Apply Rice’s theorem for recursive sets.

5. Show that the following properties of r.e. sets are r.e. Justify your answers.
   (a) $L \neq \emptyset$
   (b) $L$ contains at least 2 words.
   (c) $L$ contains some fixed word $w$.
   Hint: Apply Rice’s theorem for r.e. sets.