Multiprocessor EDF and Deadline Monotonic Schedulability Analysis

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Overview

1. question – schedulability with EDF and RM on SMP
2. key proof technique – $\lambda$-busy interval
3. summary of results
4. relation to prior results
“Dhall Effect” with EDF/DM Scheduling on MP

This is only a problem when we have some tasks with very high local utilization.
This example shows worst-case achievable multiprocessor utilization can be as bad as 1, compared to ideal value of \( m \).

Until recently, it was used as an excuse for using partitioned scheduling on multiprocessors.

However, the example depends on mixing tasks with extremely low and extremely high utilization (or ratio of compute time to deadline).

Intuitively, we know we can achieve higher utilization when we have only smaller tasks.
Demand & Load

demand of a time window =
total work $W$ that would need to be completed within the window for all the deadlines within the interval to be met

load of window of length $\Delta = W/\Delta$. 
Schedulability Test Based on Demand Analysis

Step 1:

Find a lower bound $\mu$ on the load of a window leading up to a deadline that is necessary for the deadline to be missed.

Step 2:

Find an upper bound $\beta$ on the load of such a window that can be generated by a given task set.

Step 3:

If $\beta < \mu$ the task set must be schedulable.
Lower Bound on Load of a Problem Interval

\[ \tau_k \text{ is released} \]

\[ \tau_k \text{ misses deadline} \]

\[ \text{demand} \geq x + m(d_k - x), \quad x < c_k \]

\[ \text{load} \geq m - (m - 1) \frac{c_k}{d_k} \]
We call this a *problem* interval because it ends in a missed deadline.

Since the problem job misses its deadline, the sum of the lengths of all the time intervals in which the problem job does not execute must exceed its slack time, $d_k - c_k$. 
Lower Bound on Load

Lemma The load of the interval $[t, t + d_k)$ starting in a release of $\tau_k$ and ending in a missed deadline of $\tau_k$ is

$$\frac{W}{d_k} > m(1 - \frac{c_k}{d_k}) + \frac{c_k}{d_k}$$
Upper Bound on Load

\[
\text{load} = \frac{\varepsilon + (\lfloor \frac{\Delta - d_i}{T_i} \rfloor - 1)c_i + c_i}{\Delta}
\]
The difficult part is bounding the amount of “carried in” work, \textit{i.e.} work that is released, but not completed, before the start of the interval.

If we bound this too grossly, \textit{e.g.} by $c_i$, we cannot get a useful result.
**Carried-in Load**

**Definition** The carry-in of $\tau_i$ at time $t$ is the residual compute time of the last job of task $\tau_i$ released before $t$, if any, and is denoted by the symbol $\epsilon$.

Problem: What is the best possible upper bound on carry-in?
(Mostly) Busy Interval Preceding a Missed Deadline

single processor busy window:

1. demand of the interval $= 100\%$
2. no demand is carried into the interval

multiprocessor busy window:

1. demand of the interval $> m(1 - \lambda) + \lambda$
2. demand carried into the interval is bounded by $c_k - \phi \lambda$
3. $\lambda = c_k/d_k$
We start with the interval between a missed deadline and the corresponding release time, then extend it downward until any further extension would not be $\lambda$-busy.

We call this the \textit{maximal $\lambda$-busy downward extension} of a problem interval.

This notion is the right generalization of busy window, for a multiprocessor.

(We cannot expect 100\% utilization.)
Upper Bound on EDF Demand

**Lemma** For any busy window \([t, t+\Delta]\) of \(\tau_k\) and any other task \(\tau_i\), the EDF demand \(W_i\) of \(\tau_i\) in the busy window is no greater than

\[
nc_i + \max\{0, c_i - \phi \lambda\}
\]

where \(\phi = nT_i + d_i - \Delta\),
\(\lambda = c_k/d_k\),
\(n = \left\lfloor \frac{(\Delta - d_i)}{T_i} \right\rfloor + 1\) if \(\Delta \geq d_i\), and \(n = 0\) otherwise.
The demand of a single task is the work that must be done by that task in the window for the deadline of the problem task to complete by its deadline.
Upper Bound on EDF Load

Lemma For any busy window \([t, t + \Delta]\) of \(\tau_k\) the EDF load \(W_i/\Delta\) due to \(\tau_i\) is at most \(\beta_i\), where

\[
\beta_i = \begin{cases} 
\frac{c_i}{T_i} (1 + \frac{T_i - d_i}{d_k}) & \text{if } \lambda = \frac{c_k}{d_k} \geq \frac{c_i}{T_i} \\
\frac{c_i}{T_i} (1 + \frac{T_i - d_i}{d_k}) + \frac{c_i - \lambda T_i}{d_k} & \text{if } \frac{c_k}{d_k} < \frac{c_i}{T_i}
\end{cases}
\]
This result is shown by applying the preceding bound on $W_i$ and choosing $\phi$ and $n$ to maximize $W_i/\Delta$. 
EDF Schedulablity Test

**Theorem** A set of periodic tasks $\tau_1, \ldots, \tau_n$ is schedulable on $m$ processors using preemptive EDF scheduling if, for every task $\tau_k$,

$$\sum_{i=1}^{n} \min\{1, \beta_i\} \leq m(1 - \frac{c_k}{d_k}) + \frac{c_k}{d_k}$$

where $\beta_i$ is as defined in the lemma above.
Utilization Bound

**Corollary** A set of periodic tasks $\tau_1, \ldots, \tau_n$ with maximum individual utilization $\lambda$ and with deadline equal to period, is guaranteed to be schedulable on $m$ processors using preemptive EDF scheduling if

$$\sum_{i=1}^{n} \frac{c_i}{T_i} \leq m(1 - \lambda) + \lambda$$

This was previously shown by Goossens, Funk, Baruah, using a different proof. We have it as a consequence of the more general schedulability test, for the special case where deadline equals period.

Goossens, Funk, Baruah showed that this is tight as a guaranteed utilization bound.
Upper Bound on DM Demand

Lemma For any busy window \([t, t + \Delta]\) of \(\tau_k\) and any task \(\tau_i\), the DM demand \(W_i\) of \(\tau_i\) in the busy window is no greater than

\[
nc_i - \max\{0, c_i - \phi \lambda\}
\]

where \(n = \lceil(\Delta - \delta_i)/T_i\rceil + 1\),
\(\lambda = c_k/d_k\),
\(\phi = nT_i + \delta_i - \Delta\),
\(\delta_i = c_i\) if \(i < k\), and \(\delta_i = d_i\) if \(i = k\).
Essentially the same analysis, using the $\lambda$-busy window, can be applied to fixed-priority scheduling, including deadline monotonic.
Upper Bound on DM Demand (Pictures)

$\delta = c_i$

$\delta = d_k$

$t'$

$t$

$T_i$

$T_k$

$\phi$

$d_i$

$c_i$

$c_k$

$t + \Delta$

$n-1$ $T_i$

$(n-1)$ $T_k$

head

body

tail

head

body

tail

$\Delta$

$t + \Delta$
There are two cases because (only) \( \tau_k \) must have its absolute deadline at the end of the interval. The other tasks can have absolute deadlines after that point, so long as their relative deadlines are \( \leq d_k \).
Upper bound on DM load

**Lemma** For any busy window $[t, t + \Delta)$ of $\tau_k$ the DM load $W_i/\Delta$ due to $\tau_i$, $i < k$, is at most

$$\beta_i = \begin{cases} 
\frac{c_i}{T_i} (1 + \frac{T_i - \delta_i}{d_k}) & \text{if } \lambda = \frac{c_k}{d_k} \geq \frac{c_i}{T_i} \\
\frac{c_i}{T_i} (1 + \frac{T_i - \delta_i}{d_k}) + \frac{c_i - \lambda T_i}{d_k} & \text{if } \lambda = \frac{c_k}{d_k} < \frac{c_i}{T_i}
\end{cases}$$

where $\delta_i = c_i$ for $i < k$, and $\delta_k = d_k$. 
DM Schedulability Test

**Theorem** A set of periodic tasks is schedulable on \( m \) processors using preemptive deadline-monotonic scheduling if, for every task \( \tau_k \),

\[
\sum_{i=1}^{k-1} \beta_i \leq m \left( 1 - \frac{c_k}{d_k} \right)
\]

where \( \beta_i \) is as defined in the lemma above.
RM Utilization Bound

A set of periodic tasks with maximum individual utilization $\lambda$ and deadline equal to period, is guaranteed to be schedulable on $m$ processors, $m \geq 2$, using preemptive rate monotonic scheduling if

$$\sum_{i=1}^{n} \frac{c_i}{T_i} \leq \frac{m}{2}(1 - \lambda) + \lambda$$

This improves on the result of Baruah and Goossens that a set of tasks, all with deadline equal to period, is guaranteed to be schedulable on $m$ processors using RM scheduling if $\frac{c_i}{T_i} \leq 1/3$ for $i = 1, \ldots, n$ and $\sum_{i=1}^{n} \frac{c_i}{T_i} \leq m/3$. 
Tightness of RM Utilization Bound

There exist task sets with maximum individual task utilization $\lambda$ that are not feasible with preemptive RM scheduling on $m$ processors and have utilization arbitrarily close to

$$\lambda + m \ln\left(\frac{2}{1 + \lambda}\right)$$
We conjecture that this may be the actual guaranteed RM utilization bound, though we have not been able to prove it yet.
utilization bound

lower bound
15
10
5

upper bound
20
15
10
5

\( \lambda \)
ratio of upper to lower bound
The figures show how much of a gap there is between our lower bound on the minimum achievable RM utilization and our upper bound. The first is a direct plot of the upper and lower bounds. The second shows the ratio of the upper bound to the lower bound.
Other Papers on Global MP EDF and RM Schedulability


4. S. Baruah and Joel Goossens, “Rate-monotonic scheduling on uniform multiprocessors”, UNC-CS TR02-025, University of North Carolina Department of Computer Science (May 2002).


