Exercise

Prove that $f(n) = 2n^2$ is $O(n^3)$
Exercise

• Prove that $n^2 + n$ is $O(n^2)$

$f(n) = n^2 + n \quad g(n) = n^2$

We want to find $C, n_0 > 0$ s.t.

$n^2 + n \leq C \cdot n^2 \quad \forall \ n \geq n_0$.

Let's try $C = 2$

Then we'd like to find $n_0 > 0$ s.t.

$n^2 + n \leq 2 \cdot n^2 \quad \forall \ n \geq n_0$

$n \leq n^2 \quad \forall \ n \geq n_0 \quad 1 \leq n \quad \forall \ n \geq n_0$

So set $n_0 = 1$. 
Exercise

Prove that \( n/1000 \) is \( O(n^3) \)

- We need to find \( c \) and \( n_0 \) such that \( f(n) \leq c \cdot n^3 \) for all \( n \geq n_0 \).

Let's try: \( f(n) = \frac{n}{1000} \) and \( n_0 = 1 \).

We want to show: \( \frac{n}{1000} \leq n^3 \) for all \( n \geq 1 \).

So, we set \( n_0 = 1 \).
Exercise

- Prove that $n^{2.0001}$ is $\Omega(n^2)$

We want to show that there exists $c, \eta_0 > 0$ such that $c \cdot n^2 \leq n^{2.0001}$ for $n \geq \eta_0$.

$c = 1$, $\eta_0 = 1$.

$\theta \in \mathbb{R}$.

\[ f(n) \leq g(n) \]

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Exercise

• Prove that $n^2 - 200n$ is $\Omega(n)$

Find $c, N_0 > 0$ s.t.

$$c \cdot n \leq n^2 - 200n \quad \forall n \geq N_0.$$

Try $c = 1$.

Find $N_0 > 0$ s.t.

$$n \leq n^2 - 200n \quad \forall n \geq N_0.$$

$$1 \leq n - 200 \quad \forall n \geq N_0.$$

$$n \geq 201 \quad \forall n \geq N_0$$

$N_0 = 201$. 
Exercise

- Prove that $\frac{n^2}{2} - 2n$ is $\Theta(n^2)$.

We want to find $c_1, c_2, n_0 > 0$ such that

$$c_1 g(n) \leq f(n) \leq c_2 g(n) \quad \forall n \geq n_0.$$

We want to find $c_1$ and $n_0$ such that

$$c_1 \frac{n^2}{2} - 2n \leq f(n) \leq n^2 / 4 \quad \forall n \geq n_0'.

Let's try $c_1 = \frac{1}{4}$

$$\frac{n^2}{2} - 2n \leq \frac{n^2}{4} \quad \forall n \geq n_0'.

8 \leq n

$n_0' = 8$

We want to find $c_2$ and $n_0$ such that

$$\frac{n^2}{2} - 2n \leq c_2 g(n) \quad \forall n \geq n_0.

Try $c_2 = \frac{1}{4}$

$$\frac{n^2}{2} - 2n \leq \frac{n^2}{4} \quad \forall n \geq n_0.

8 \leq n

$n_0 = \max\{8, -4\} = 8$
Exercise

Prove that \( n^2 / \log n = o(n^2) \).
Exercise

Prove that $n^2/1000 \neq o(n^2)$.

We want to find a value of $n$ such that $f(n) = \frac{n^2}{1000}$ is not $o(n^2)$. Let's try $c = 100$, $n = \frac{1000}{c}$.
Exercise

Prove that $n^2.001 = \omega(n^2)$. 

$$g(n) = \begin{cases} 
1 & \text{if } n \leq 1000 \\
\frac{e^n}{n^2} & \text{otherwise} 
\end{cases}$$ 

$$\lim_{n \to \infty} \frac{g(n)}{\ln(n)} = \infty$$
Exercise

1. Prove that $n^2 \lg n = \omega(n^2)$

2. Calculate $\lim_{n \to \infty} \frac{n^2}{\log n}$
Exercise

• Prove that $n^2 \neq \omega(n^2)$

By way of contradiction, assume that

$m^2 = \omega(n^2)$

Then $\forall C \exists N$ s.t. $C \cdot g(n) < f(n)$ $\forall n \geq N_0$

So, $C \cdot n^2 < n^2$ $\forall n \geq N_0$.

In particular, if $C = 1$,

$n^2 < n^2$ $\forall n \geq N_0$.
Properties: Exercise 2

- Prove that if \( f = \Omega(h) \) and \( g = \Omega(h) \) then \( f + g = \Omega(h) \).

Since \( f(n) \in \Omega(h(n)) \) \( \exists \ c > 0 \) and \( n_0 > 0 \) s.t. \( A \ n \geq n_0 \) \( f(n) \leq c n \).

Since \( g(n) \in \Omega(h(n)) \) \( \exists \ c' > 0 \) and \( n_0' > 0 \) s.t. \( c' h(n) \leq g(n) + A \ n \geq n_0' \).

We want to show that \( \exists \ c'', n_0'' > 0 \) s.t.
\[
  c'' h(n) \leq f(n) + g(n) + A \ n \geq n_0''.
\]

So set \( c'' = c + c' \) and \( n_0'' = \max(n_0, n_0') \).
Properties: Exercise 1

1. Prove that if $f = O(g)$ and $g = O(h)$ then $f = O(h)$.

Since $f = O(g)$, we have $f(n) = c_1 g(n)$ for some $c_1 > 0$ and all $n \geq n_1$. Since $g = O(h)$, we have $g(n) = c_2 h(n)$ for some $c_2 > 0$ and all $n \geq n_2$. Therefore, we can combine these two inequalities to get $f(n) = c_1 c_2 h(n)$ for all $n \geq \max(n_1, n_2)$. Thus, $f = O(h)$.