COP4531 - Fall 2015: Assignment 5

Tuesday, November 24, 3:35PM.
Please note that no assignments will be accepted late.

Problem 1
Prove that the following problems are in NP.

A. Given a data set in Euclidean space, is there a clustering of this data with \( k \)-means cost at most \( c \)?

B. Given a graph, determine whether there is a path in the graph that visits each node exactly ones.

C. Given an unweighted graph, and a pair of vertices \( a \) and \( b \), does there exist an odd length path between \( a \) and \( b \)?

Problem 2
Assume that problem \( A \) is some NP-complete problem.

A. Let \( B \) be the independent set problem. Is it true that \( A \leq_p B \)? Justify your answer.

B. Let \( C \) be the set cover problem. Is it true that \( C \leq_p A \)? Justify your answer.

C. Let \( D \) be some problem in \( NP \). Is it true that \( D \leq_p A \)? Justify your answer.

Problem 3

A. Given two problems \( X \) and \( Y \), if \( X \leq_p Y \), does it imply that \( Y \leq_p X \)? Prove your answer.

B. Let \( X \) and \( Y \) be problems in \( NP \). If there is a polynomial time solution to \( X \), does it mean that there is a polynomial time solution to \( Y \)? Justify your answer.
Problem 4

Consider a modified queue, with the following operations:

- INIT(S): Create an empty modified queue.
- ENQUEUE(S, x): Add x to the end of the modified queue.
- DEQUEUE(S): Delete and return the first element in the modified queue.
- REMOVE(S, k): Delete the first k elements from the modified queue.
- EMPTY(S): Delete all elements from the modified queue.

We implement a modified queue as a double linked list (maintaining a pointer to the front and back element).

A. What is the worst-case running time of each of these operations?

B. What is the amortized running time of m of these operations? Justify your answer.

Problem 5

Consider the stable marriage problem as applies to 3 men (Bob, Steve, and Alex) and 3 women (Alice, Karen, and Mia).

The following are their preference lists:

- Alice: Bob, Steve, Alex
- Karen: Steve, Alex, Bob
- Mia: Steve, Alex, Bob
- Bob: Karen, Mia, Alice
- Steve: Mia, Karen, Alice
- Alex: Alice, Mia, Karen

A. Prove that the following matching is not stable: (Steve, Alice), (Alex, Karen), (Bob, Mia)

B. Find a stable matching for this instance of the stable marriage problem. Explain why it is stable.