Performance Modeling of Bufferless WDM Packet Switching Networks with Wavelength Conversion

Zhenghao Zhang and Yuanyuan Yang
Department of Electrical & Computer Engineering, State University of New York, Stony Brook, NY 11794, USA

***Abstract*— In this paper we study the performance of bufferless optical WDM packet switching networks with various wavelength conversion degrees. We first introduce an optimal scheduling algorithm that maximizes the network throughput in this type of network. We then derive a novel analytical model to evaluate the network performance in terms of packet loss probability under the scheduling algorithm for Bernoulli traffic. Our model is the first accurate analytical model for bufferless WDM packet switching with variable conversion degrees and can be used to quantitatively determine the minimum wavelength conversion degree required for a certain traffic load. We also conducted simulations to validate the analytical model. Both the analytical and simulation results reveal that limited range wavelength conversion can achieve almost the same network performance as full wavelength conversion.**

I. INTRODUCTION

Many emerging computing/networking applications, such as data-browsing in the world wide web, video conferencing, video on demand, E-commerce and image distributing, require very high network bandwidth often far beyond that today's high-speed networks can offer. Optical networking is a promising solution to this problem because of the huge bandwidth of optics: a single fiber has a bandwidth of nearly 50 THz. To fully utilize the bandwidth, a fiber is divided into a number of independent channels, with each channel on a different wavelength. This is referred to as *wavelength-division-multiplexing (WDM)*.

Several different technologies have been developed for transmitting data over WDM [4], such as broadcast-and-select, wavelength routing, optical packet switching, and optical burst switching. Broadcast-and-select and wavelength routing has been extensively studied. Optical packet switching and burst switching, especially optical packet switching, although still in their research phase, are attracting more and more interests as it may offer better flexibility and better exploitations of the bandwidth. In this paper, we will focus on WDM packet switching networks.

In a WDM optical packet switching network, data packet is modulated on a specific wavelength and may travel several hops before reaching the destination. In each hop, a switching network (or simply a switch) is used to direct the packet to the correct output fiber link. Output contention occurs when some packets on the same wavelength are destined for one output fiber. There are three ways to combat output contention: buffering, deflection routing and exploiting the wavelength domain [4]. Buffering is to use fiber delay lines to delay the packets for a certain amount of time to avoid the contention period, however this may add to the cost and size of the switch since fiber delay lines are costly and bulky. Deflection routing is to send the contending packet to some other output link which may or may not have a route to the destination. By doing so, the packet is not dropped but the end-to-end delay may be long and the packets arrived at the destination may not be in a correct order. In this paper we study the third method, exploiting the wavelength domain, which is to convert the wavelengths of the contending packets to some idle wavelengths (if there are any) on the destination output fiber such that the packets can still be transmitted. The translation of wavelengths is achieved by using a wavelength converter which converts a signal on one wavelength to another. There are *full range wavelength converters* which can convert a wavelength to any other wavelength in the optical system. However, this type of wavelength converter is quite difficult and expensive to implement due to technological limitations [8], [3]. A realistic all-optical wavelength converter may only be able to convert a given wavelength to a set of adjacent wavelengths. The number of the wavelengths in this set is called *conversion degree*. This type of wavelength converter is called *limited range wavelength converter*, and will be our main focus in this paper.

Recently, limited range wavelength conversion in wavelength routed WDM networks (similar to circuit switching electronic networks) has been extensively studied, see, [3] [8], for a survey. [6], [7] studied WDM packet switching networks with full range wavelength conversion, and especially in [7], unbuffered WDM switches were shown to have an acceptable packet loss probability (10^{-10}) only when the number of wavelengths on a fiber is large. WDM packet switches with limited range wavelength conversion have been studied by [5] in which limited range wavelength conversion was shown to have a close performance to that of full range wavelength conversion. However, the results in [5] were obtained by simulations only and the packet loss probabilities of the tested switching networks are all greater than 10^{-7}, while the generally accepted packet loss probability of a switching network is 10^{-10}. In this paper we will give an accurate analytical model to study the performance of WDM packet switching networks with limited range wavelength conversion under Bernoulli traffic. To the best of our knowledge, this is the first analytical model for such networks. Our model can be used to quantitatively determine the minimum conversion degree required under a certain traffic load such that the packet loss probability is below 10^{-10}.

Note that when output contention occurs, to maximize network throughput, a scheduling algorithm that selects the largest group of contention-free packets is needed. Thus, we first introduce First Available Algorithm as such a scheduling algorithm. Then we give a recursive method to find the probability mass function (p.m.f.) of the number of used wavelengths per output fiber, by which we derive the packet loss probability of the network. Since First Available Algorithm maximizes network throughput, our analytical model reveals the maximum network capacity. We also conducted simulations to validate the analytical model and the results show that the analytical model is very accurate. Both of our analytical results and simulations show that the packet loss probability can be controlled under (10^{-10}) if the traffic load is below a certain level determined by the network parameters. The results also show that limited range wavelength conversion has a close packet loss probability to full range wavelength conversion.

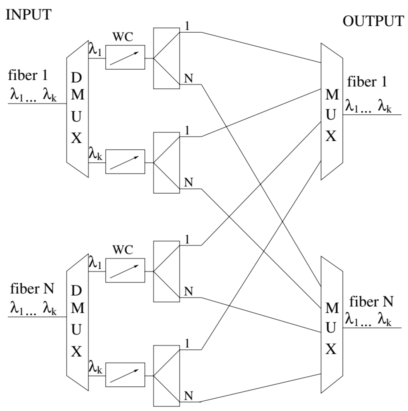

Fig. 1. A wavelength convertible WDM optical switching network.

II. PRELIMINARIES

A. Wavelength Conversion

As mentioned earlier, in a WDM optical switching network with limited range wavelength conversion, an input wavelength may be converted to a set of adjacent outgoing wavelengths. We define the set of these outgoing wavelengths as the *adjacency set* of this input wavelength. The cardinality of the adjacency set is the *conversion degree* of this wavelength. We assume that the adjacency set of any wavelength λ_i for $i \in \{0, 1, \ldots, k-1\}$ is interval $[u, v]$, where $u = \max\{0, i-d\}$ and $v = \min\{k-1, i+d\}$. Note that under this assumption wavelengths may have different conversion degrees. The wavelengths in the middle have a larger degree of $2d+1$, while the wavelengths near the ends have smaller conversion degrees, with the smallest one being $d+1$. In the following, we define the conversion degree of a wavelength converter as $2d+1$, the maximum conversion degree of all wavelengths. To facilitate the discussion, we would mainly use d which is defined as *conversion distance* as a measure of the conversion ability of a wavelength converter, as all wavelengths have the same conversion distance.

B. The Network Model

The WDM switching network we consider has N input fibers and N output fibers, and on each fiber there are k wavelengths. Thus, there are a total of Nk input wavelength channels and Nk output wavelength channels. Any input wavelength channel can be connected to any output fiber. In addition, there are limited range wavelength converters equipped for each input wavelength channel and as a result, an input wavelength channel can be connected to any of its adjacent channels on an output fiber. Fig. 1 shows such an WDM interconnect. It can be seen from the figure that an input fiber is first fed into a demultiplexer, where different wavelength channels are separated from one another. The signals are then fed into wavelength converters to be converted to proper wavelengths. The output of the wavelength converter can then be connected to one of the N output fibers, controlled by the splitter. Before each output fiber there is an optical combiner to multiplex the signals on different wavelengths into the fiber. Apparently, it is required that all the signals to an optical combiner must be on different wavelengths.

As in [5], we assume that the WDM optical switching network is operated in a synchronous mode, in which optical packets or cells arrive at the network at the beginning of time slots. We consider the case where all packets are of the same priority and of same length. Because optical buffers are currently made of fiber delay lines and are still very expensive [1], we also assume that there are no buffers in the WDM optical switch. The traffic pattern considered in this paper is unicast, i.e., each packet is destined for only one output fiber. The packet does not specify which wavelength channel on the destination fiber it should be connected to, and can be assigned to any free wavelength channel accessible to it.

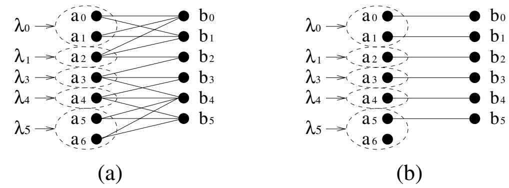

Fig. 2. The request graph and its maximum matching when request vector is [2,1,0,1,1,2] in a 6-wavelength switching network with conversion distance 1. (a) Request graph. (b) Maximum matching.

III. OPTIMAL CONTENTION-FREE SCHEDULING ALGORITHM

A. Problem Formalization

Under the assumptions made in Section II, the packets arrived at the switch in one time slot can be partitioned into N subsets according to their destinations. The decision of accepting a packet or not in one subset does not affect the decisions in other subsets. This suggests that the scheduling algorithm can be run independently for each output fiber. The input to this scheduling algorithm is the packets destined to this fiber. The output of the algorithm is the decision whether a packet can be transmitted, and if yes, which wavelength channel it is assigned to.

We depict the relationship between the packets destined for an output fiber and the available wavelength channels on that output fiber by a bipartite graph, called *request graph*. We use A to represent the set of the left side vertices and B for the right side vertices. Each left side vertex represents a packet and each right side vertex represents an output wavelength. The right side vertices are depicted according to their wavelength indexes, for example, λ_0 is above λ_1, λ_1 is above λ_2 and so on. The left side vertices are also depicted according to their wavelength indexes, with packets on the same wavelength in an arbitrary order.

There is an edge connecting a left side vertex a and a right side vertex b if the wavelength of packet a can be converted to output wavelength b. We also define the *request vector* which is a $1 \times k$ row vector, with the i_{th} element representing the number of packets arrived on wavelength λ_i. Fig. 2(a) shows the request graph when the request vector is $R = [2, 1, 0, 1, 1, 2]$.

In a request graph G, let E be the set of edges. Any wavelength assignment can be represented by a subset of E, E', where edge $ab \in E'$ if wavelength channel b is assigned to packet a. Under unicast traffic, any packet needs only one output channel. Also, an output channel can be assigned to only one packet. It follows that the edges in E' are vertex disjoint, since if two edges share a vertex, either one packet is assigned two wavelength channels or one wavelength channel is assigned to two packets. Thus, E' is a *matching* in G. For a given set of packets, to maximize network throughput, we should find a maximum matching in the request graph. The maximum matching for Fig. 2(a) is shown in Fig.

TABLE 1

FIRST AVAILABLE ALGORITHM

```
First Available Algorithm
 for i := 0 to k − 1 do
        let a_j be a vertex in A adjacent to b_i
        not matched yet and is with the smallest index
        if no such a_j exists
              MATCH[i] := Λ
        else
              MATCH[i] := j
        end if
 end for
```

2(b). In this example, not all left side vertices can be matched because there are seven vertices on the left (packets) while there are only six vertices on the right (available wavelength channels).

B. First Available Algorithm

Since finding the largest group of contention-free packets is equivalent to finding a maximum matching in the request graph, we will describe the algorithm as a maximum matching algorithm for bipartite graphs.

As discussed in [9], in the request graph we consider, the adjacency set of a right side vertex b_i is an interval and can be represented as $[begin(b_i), end(b_i)]$ where $begin(b_i)$ and $end(b_i)$ are nonnegative integers. For two right side vertices b_i and b_j, if $i < j$, $begin(b_i) \leq begin(b_j)$ and $end(b_i) \leq end(b_j)$. For a bipartite graph with these two properties, First Available Algorithm in Table 1 given by [9] can be used to find a maximum matching. The input to this algorithm is: (1) The set of left side vertices A and the set of right side vertices B; (2) For each right side vertex b, the set of vertices adjacent to it represented by interval $[begin(b), end(b)]$. The output of the algorithm is array $MATCH[]$. $MATCH[i] = j$ means that the i_{th} right side vertex is matched to the j_{th} left side vertex. $MATCH[i] = \Lambda$ if the i_{th} right side vertex is not matched to any left side vertex.

This algorithm checks the right side vertices from the top to the bottom. A right side vertex b_i is matched to the first free adjacent left side vertex if such vertex exists; otherwise it is left unmatched. Fig. 2(b) is the maximum matching found by this algorithm for Fig. 2(a). [9] proved that that First Available Algorithm finds a maximum matching in a request graph. The time complexity of this algorithm is $O(k)$, since the loop is executed k times which is the number of right side vertices or the number of wavelengths per fiber.

IV. THE ANALYTICAL MODEL

In this section we present an analytical model for WDM switching networks with limited wavelength conversion. We are particularly interested in finding how network performance changes with regarding to the conversion degree. The network performance is measured by the *packet loss probability* which is defined as the ratio of the number of rejected packets over all arrived packets. It is a function of network size N, the number of wavelengths k, conversion distance d and arrival rate ρ, and is also affected by the scheduling algorithm. From the previous section we know that maximum network throughput or the minimum packet loss probability can be achieved by adopting First Available Algorithm. Hence we will derive our analytical model under this algorithm.

The following assumptions are made about the traffic:

- Each packet holds for one time slot.
- Input channels (wavelengths) are independent of each other, i.e., at one time slot, the probability that there is a packet at an input channel is independent of other input channels.
- The arrival at each input channel is Bernoulli with parameter ρ ($0 \leq \rho \leq 1$), i.e., the probability that there is a packet at one input channel at one time slot is ρ and independent of other time slots.
- The destination of a packet is uniformly distributed over all N output fibers.

Under these assumptions, at one output fiber, the request vector $R = [r_0, r_1, \ldots, r_{k-1}]$ at one time slot is independent of the the request vector at other time slots. Moreover, since the components of the request vector come from different wavelengths, they are also independent of each other and follow the same distribution. We use $p_r()$ to denote the p.m.f. of the components in the request vector. $p_r(i)$ is the probability that there are i packets on a particular wavelength destined for this output fiber. We have

$$p_r(i) = \binom{N}{i} \left(\frac{\rho}{N}\right)^i \left(1 - \frac{\rho}{N}\right)^{N-i}, \; i = 0, 1, \ldots, N. \quad (1)$$

That is, the number of packets on each wavelength is a Binomial random variable with parameter $(N, \frac{\rho}{N})$. In the following we will use $P_r()$ as the c.d.f. of the components of the request vector. $P_r(j)$ is the probability that there are no more than j packets on a particular wavelength destined for this output fiber.

Let U^i be the number of used output wavelength channels on output fiber i in a time slot. U^i is a discrete random variable taking values from 0 to k. At one time slot, U^i, $1 \leq i \leq N$, are not independent of each other. However, based on the assumptions on the uniformity of the traffic, they follow the same distribution. If we consider a long time period, the number of packets have been transmitted through one output fiber should be the same as all other output fibers. Hence, if we know the throughput at one output fiber, we know the throughput of the entire network. Therefore it is sufficient to consider one output fiber, and in the following we omit the superscript of U^i and simply write it as U. We use $p_U(i)$, $0 \leq i \leq k$ to denote the p.m.f. of U. The average number of used wavelengths at an output fiber is the mean value of U, $E(U) = \sum_{i=0}^{k} i p_U(i)$. The average number of used output wavelengths in the network is $NE(U)$, and the average number of arrived packets is $Nk\rho$. It follows that the packet loss probability can be written as

$$P_{loss} = 1 - E(U)/k\rho \quad (2)$$

In the following we give a recursive method to calculate $p_U(i)$ analytically. First we introduce the definition of random variable $U_{m,n}$, where m and n are positive integers. $U_{m,n}$ is defined as the number of used output wavelengths from λ_{k-n} to λ_{k-1}, under the condition that output wavelengths λ_{k-n} to λ_{k-1} are only assigned to packets on wavelengths λ_{k-m} to λ_{k-1}, and packets on wavelengths λ_{k-m} to λ_{k-1} are only assigned to output wavelengths λ_{k-n} to λ_{k-1}, when First Available Algorithm is used. $U_{m,n}$ is a random variable takes values from 0 to n and we use $p_U(m, n, i)$ to denote its p.m.f. for $0 \leq i \leq n$. Fig. 3 illustrates the definition of $U_{m,n}$. Clearly, when $m = n = k$, $U_{k,k}$ is simply U. Thus, the p.m.f. of $U_{k,k}$ can be used to evaluate network performance.

Let's first consider $U_{m,n}$ for $m \geq d + 2$ and $m - d \leq n \leq m + d$. Let $T = n - m + d + 1$ for these m and n. T is the

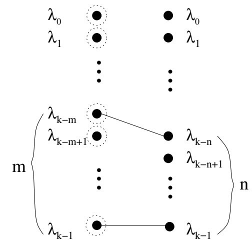

Fig. 3. $U_{m,n}$ is the number of used output wavelengths from λ_{k-n} to λ_{k-1}, knowing that output wavelengths λ_{k-n} to λ_{k-1} are only assigned to packets on wavelengths λ_{k-m} to λ_{k-1}, and packets on wavelengths λ_{k-m} to λ_{k-1} are only assigned to output wavelengths λ_{k-n} to λ_{k-1}.

number of output wavelengths adjacent to the first of the m input wavelengths (λ_{k-m}). We have the following recursive relations:

$$p_U(m,n,i) = \tag{3}$$
$$\begin{cases} \sum_{j=0}^{i} p_r(j)p_U(m-1,n-j,i-j), & 0 \le i < T \\ \sum_{j=0}^{T} p_r(j)p_U(m-1,n-j,i-j)+ \\ \qquad [1-P_r(T)]p_U(m-1,m-d-1,i-T), & T \le i \le n \end{cases}$$

Recall that $p_r()$ and $P_r()$ are the p.m.f and c.d.f. of random variable r, respectively. Equation (4) can be derived by conditioning on r_{k-m} which is the number of packets on the first wavelength (λ_{k-m}). $p_U(m,n,i)$ can be written as

$$p_U(m,n,i) = \sum_j Prob.(U_{m,n} = i \mid r_{k-m} = j)Prob.(r_{k-m} = j)$$

Note that $Prob.(r_{k-m} = j)$ is simply $p_r(j)$, the probability that there are j packets on wavelength λ_{k-m}. In the following we derive the conditional probability $Prob.(U_{m,n} = i \mid r_{k-m} = j)$. There are T output wavelengths adjacent to λ_{k-m}. We claim that given there are j packets on λ_{k-m}, if $j \le T$, the first j output wavelengths, λ_{k-n} to $\lambda_{k-n+j-1}$, will be assigned to incoming packets on λ_{k-m}; otherwise $j > T$ and the first T output wavelengths, λ_{k-n} to $\lambda_{k-n+T-1}$, will be assigned to incoming packets on λ_{k-m}. This is because by the definition of $U_{m,n}$, none of the packets on λ_{k-m} is assigned to output wavelengths with smaller indexes than λ_{k-n}, and output wavelengths λ_{k-n},λ_{k-n+1}, λ_{k-n+2}, $\ldots$ are not assigned to packets on wavelengths with smaller indexes than λ_{k-m}. Therefore, when First Available Algorithm is checking output wavelengths λ_{k-n}, λ_{k-n+1}, λ_{k-n+2}, $\ldots$, $\lambda_{k-n+j-1}$ (or $\lambda_{k-n+T-1}$ if $j > T$), they will all be assigned to packets on λ_{k-m}, as these are the "first available" packets. There are only T output wavelengths accessible to the packets on λ_{k-m}. Thus at most the first T of them can be assigned.

Now if $i < T$, if $j \le i$, $Prob.(U_{m,n} = i \mid r_{k-m} = j)$ is the probability that among the rest of the output wavelengths, λ_{k-n+j} to λ_{k-1}, $i-j$ wavelengths are used. Note that these output wavelengths cannot be assigned to packets on wavelengths with smaller indexes than λ_{k-m+1} and packets on wavelengths from λ_{k-m+1} to λ_{k-1} are only assigned to these output wavelengths. Therefore by definition, $Prob.(U_{m,n} = i \mid r_{k-m} = j)$ is simply $p_U(m-1,n-j,i-j)$. If $j > i$, $Prob.(U_{m,n} = i \mid r_{k-m} = j)$ is zero, since among λ_{k-n} to λ_{k-1} there must be j wavelengths used. Similarly we can find the conditional probability for $i \ge T$: if $j \le T$, $Prob.(U_{m,n} = i \mid r_{k-m} = j) = p_U(m-1,n-j,i-j)$; otherwise $Prob.(U_{m,n} = i \mid r_{k-m} = j) = p_U(m-1,m-d-1,i-T)$.

Also, for $2 \le m \le d+1$, $p_U(m,n,i)$, where $1 \le n \le m+d$ and $0 \le i \le n$ can be obtained by conditioning on r_{k-m}:
$p_U(m,n,i) =$

$$\begin{cases} \sum_{j=0}^{i} p_r(j)p_U(m-1,n-j,i-j), & 0 \le i < n \\ \sum_{j=0}^{n-1} p_r(j)p_U(m-1,n-j,i-j) + [1-P_r(n-1)], & i = n \end{cases} \tag{4}$$

Finally, consider $m = 1$ and $1 \le n \le d+1$. $p_U(1,n,i)$ is the probability that there are i matched wavelengths among λ_{k-n} to λ_{k-1}, knowing that these wavelengths can only be assigned to packets on wavelength λ_{k-1}, and that packets on wavelength λ_{k-1} are not assigned to other output wavelengths. Now incoming wavelength λ_{k-1} can be converted to any wavelength in the range λ_{k-n} to λ_{k-1}. It follows that

$$p_U(1,n,i) = \begin{cases} p_r(i) & 0 \le i < n \\ 1-P_r(n-1) & i = n \end{cases} \tag{5}$$

since if there are i packets, if $i < n$, i wavelengths will be used; if $i \ge n$, n wavelengths will be used.

To find the p.m.f. for $U_{k,k}$, start with random variables when $m = 1$: $U_{1,1}$, $U_{1,2}$, $\ldots$, $U_{1,d+1}$ whose p.m.f. can be found by Equation (5). Then use Equation (4) and the p.m.f. of $U_{1,1}$, $U_{1,2}$, $\ldots$, $U_{1,d+1}$ to find the p.m.f. for the random variables when $m = 2$: $U_{2,1}$, $U_{2,2}$, $\ldots$, $U_{2,d+2}$. Repeatedly applying Equation (4) and the p.m.f found in the previous step to obtain the p.m.f. for random variables when $m = 3$, $m = 4$, $\ldots$, until $m = d+1$. Then use Equation (4) to obtain the p.m.f. for random variables for larger m until p.m.f. for $U_{k,k}$ is found. Recall that $U_{k,k}$ is exactly the random variable of the number of used wavelengths per output fiber, or U.

V. Finding minimum conversion distance

In this section we illustrate how to use our analytical model to efficiently find system parameters in networks designs. We will answer the question: Given a traffic load, what is the minimum conversion distance required to make the packet loss probability less than 10^{-10}? It is worth pointing out that only with an analytical model is it possible to find this minimum conversion distance, as repeating simulations to find system parameters accurately when packet loss probability is so low will need a huge amount of computing time.

First, given network size N, number of wavelengths k, and conversion distance d, there must be an upper limit on traffic load ρ such that the packet loss probability is less than 10^{-10}. Denote this upper limit by ρ^d_{max}. The following method can be used to find ρ^d_{max} for any d. Start with $\rho = 0$, calculate the packet loss probability. If it is under 10^{-10}, increment ρ by Δ where Δ is a small value, and calculate the packet loss probability again. Keep on incrementing ρ until the packet loss probability exceeds 10^{-10}. Then $\rho - \Delta$ is the ρ^d_{max} for this d. We can build a table of ρ^d_{max} for all possible conversion distances. Now given a traffic load ρ, we can simply look through this table to find the minimum d whose ρ^d_{max} is no less than ρ. This d will be the minimum conversion distance.

We define ρ_f as the maximum load to a network with full range wavelength conversion such that the packet loss probability is under 10^{-10}. The packet loss probability of such a WDM switch is

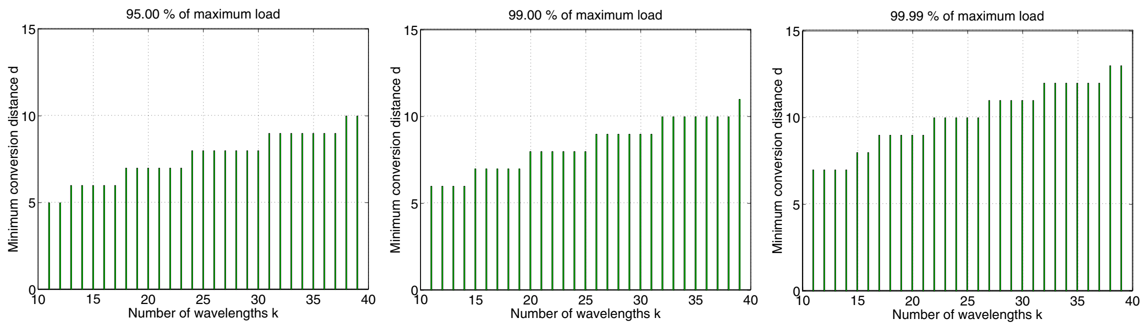

Fig. 4. Minimum conversion distance when arrival rates are 95.00% of ρ_f, 99.00% of ρ_f and 99.99% of ρ_f, respectively.

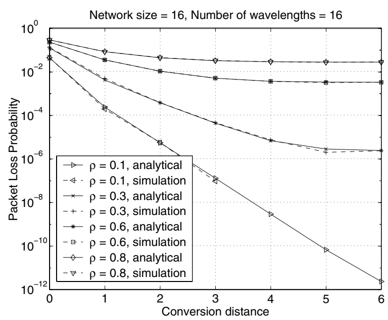

Fig. 5. Packet loss probability as a function of conversion distance, where network size is 16 and the number of wavelengths is 16.

$$\frac{\sum_{i=k+1}^{Nk}(i-k)p_f(i)}{\sum_{i=0}^{Nk} ip_f(i)} \tag{6}$$

where $p_f(i) = \begin{pmatrix} Nk \\ i \end{pmatrix} \left(\frac{\rho}{N}\right)^i \left(1-\frac{\rho}{N}\right)^{Nk-i}, \; i = 0, 1, \ldots, Nk.$ ρ_f can be found numerically using (6). ρ_f will be grater than any ρ_{max}^d and we will refer to ρ_f as the "maximum load".

We found that when the arrival rate is less than but very close to ρ_f, the packet loss probability can be controlled under 10^{-10} by using limited range wavelength converters. Fig. 4 shows the minimum conversion distance required to control packet loss probability under 10^{-10} when the arrival rates are 95.00% of ρ_f, 99.00% of ρ_f and 99.99% of ρ_f, respectively. We can see that minimum conversion distance is usually only a fraction of the total number of wavelengths k. For example, when $k = 36$, to achieve 99.00% of ρ_f, the minimum conversion distance is only 10, and the conversion degree is 21. This suggests that to achieve an almost best possible performance (to control packet loss probability below 10^{-10} under an arrival rate very close to ρ_f), less costly limited range wavelength converters can be used. It can also be considered that when the conversion distance is sufficiently large, increasing the conversion distance can only gain a very small amount of benefit in terms of maximum allowable load.

VI. Simulation Results

In this section we present simulation results to validate our analytical model. The traffic is uniform Bernoulli with arrival rate ρ. We conducted simulations on networks of various sizes, numbers of wavelengths and conversion degrees. Due to the limit of space we only show in Fig. 5 the packet loss probability as a function of conversion distance d where network size $N = 16$, number of wavelengths $k = 16$ and arrival rates ρ is 0.1, 0.3, 0.6 and 0.8, respectively. First, we notice that the analytical results and the simulation results are very close. The reason is that our analytical model is accurate under the assumptions made on the traffic. Secondly, as expected, the packet loss probability decreases as conversion distance increases for all arrival rates. An interesting fact is that as the conversion degree increases, the improvement or the decreasing rate of the packet loss probability decreases. In other words, the packet loss probability decreases more slowly as the conversion degree increases. Similar facts have been observed in the previous section when we were searching for the minimum conversion distance.

VII. Conclusions

In this paper we have studied the performance of bufferless WDM optical packet switching networks with limited range wavelength conversion. We first introduced an optimal contention-free scheduling algorithm under which network throughput can be maximized. We then gave an analytical model for network packet loss probability under uniform Bernoulli traffic. We also conducted simulations to validate the analytical model. The results show that our analytical model is accurate under the assumptions made. We have used the analytical model to find the minimum conversion distance required to make packet loss probability less than 10^{-10} under a given traffic load. Both the analytical and simulation results reveal clearly that limited range wavelength conversion has a close network performance to full range wavelength conversion.

References

[1] D. K. Hunter, M. C. Chia and I. Andonovic "Buffering in optical packet switches," *J. Lightwave Technology*, vol.16, no.12, pp.2081-2094, 1998.

[2] M. Kovacevic and A. Acampora, "Benefits of wavelength translation in all-optical clear-channel networks," *JSAC*, vol.14, no.5, pp.868 -880, 1996.

[3] T. Tripathi and K.N. Sivarajan, "Computing approximate blocking probabilities in wavelength routed all-optical networks with limited-range wavelength conversion," *IEEE INFOCOM*, vol. 1, pp. 329-336, 1999

[4] L. Xu, et al., "Techniques for optical packet switching and optical burst switching," *IEEE Comm. Mag.*, pp. 136 - 142, Jan. 2001.

[5] G. Shen, et al., "Performance study on a WDM packet switch with limited-range wavelength converters," *IEEE Comm. Letters* , vol. 5, no. 10, pp. 432-434, Oct. 2001

[6] S.L. Danielsen, B. Mikkelsen, C. Joergensen, T. Durhuus, and K.E. Stubkjaer, "WDM packet switch architectures and analysis of the influence of tunable wavelength converters on the performance," *J. Lightwave Technology*, vol. 15, no. 2, pp. 219-227, Feb. 1998.

[7] V. Eramo and M. Listanti, "Packet loss in a bufferless optical WDM switch employing shared tunable wavelength converters," *J. Lightwave Technology*, vol. 18, no. 12, pp. 1818-1833, Dec. 2000.

[8] R. Ramaswami and G. Sasaki, "Multiwavelength optical networks with limited wavelength conversion," *IEEE/ACM Trans. Networking*, vol. 6, pp. 744-754, Dec. 1998.

[9] Z. Zhang and Y. Yang, "Distributed scheduling algorithms for wavelength convertible WDM optical interconnects," *IEEE IPDPS '03*, April, 2003.

