Homework One

1. (10 points) Calculate the channel capacity if the bandwidth is 1 MHz and signal to noise ratio (S/N) is 127 according to the Shannon's theorem. (Easy, please get it right!)

2. (10 points) We say a sine wave is at frequency f Hz if it is can be written as $cos(2\pi ft)$. Let $x(t) = cos(3\pi ft)cos(\pi ft)$. If $x(t)$ is sent to a low-pass filter with cut-off frequency of $3f/2$ Hz, meaning that all signals on frequency $3f/2$ Hz or higher will be filtered out, what is the output of the filter?

3. (10 points) Consider a communication link. Suppose the receiver receives $y = x + n$, where x is -1 (representing bit 0) and 1 (representing bit 1) with equal probability and n is the noise. Based on the value of y, a detector outputs either 0 or 1.

   (a) If $\mid n \mid \leq 0.5$, how would you design the detector?

   (b) If the noise takes value from $\{-1.5, -0.5, 0.5, 1.5\}$ with probability $\{0.4, 0.1, 0.1, 0.4\}$, respectively, how would you design the detector? What is the probability that your detector makes a mistake?

4. (10 points) Consider the basic wireless communication system, where the sender sends $I(t)cos(2\pi ft)$ and the receiver multiplies the received waveform with its local carrier and then sends the signal to a low pass filter (which will pass the baseband signals but not signals on higher frequencies).

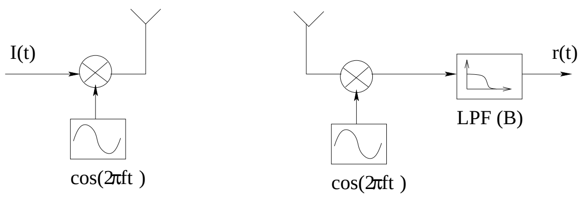

Figure 1: Problem 4.

   (a) If the receiver's local carrier is not synchronized with the sender's carrier and is $cos(2\pi ft + \phi)$ where ϕ is the phase difference, please write down the signal after the low pass filter. If $\phi = \pi/2$, can the receiver receive any baseband signal?

   (b) What if the receiver uses two receiving channels, one multiplying the input by $cos(2\pi ft + \phi)$ and the other by $sin(2\pi ft + \phi)$? If $\phi = \pi/2$, can the receiver receive any baseband signal?

5. (10 points) Suppose the 3G carrier is on 1.8 GHz and the Wi-Fi carrier is on 2.4 GHz. If a 3G phone transmits signals, will a Wi-Fi receiver receive any baseband signal? Why? Assume the bandwidth of the Wi-Fi signal is 20 MHz.

6. (10 points) If to minimize the bit error ratio, which assignment of bits to the QPSK constellation makes more sense? Why?

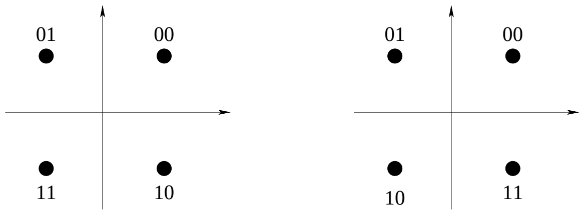

Figure 2: Problem 5.

7. (10 points) Suppose a Wi-Fi carrier is on 2.4 GHz and there are two paths from the sender to the receiver of same signal strength with length 12.5 m and 12.5625 m, respectively. Assume the bandwidth of the Wi-Fi signal is 20 MHz. Will the antenna of the receiver receive any signal at 2.4 GHz? Will the receiver receive any baseband signal?

8. (10 points) The (7,4) Hamming code we discussed in class can correct 1 bit error in a codeword. Assume:

   - bit errors occur independently in the stream of bits,
   - any codeword with more than 1 bit error cannot be corrected and the number of errors after the decoding is 7 (worst case).

   Calculate the Bit Error Ratio (BER) for the data bits protected by the Hamming code if the raw BER of the channel is $p = 0.0001$.

9. (10 points) The (7,4) Hamming code we discussed in class has minimum distance 3. Using this fact, prove that no two rows in the parity checking matrix H are the same. (Hint: Recall that the minimum distance is the minimum weight of codewords, and any codeword multiplied with H is 0.)

10. (10 points) The (7,4) cyclic we discussed in class has generator polynomial $g(x) = x^3 + x + 1$. Find the code polynomial for $m(x) = x^2+x$ when $C(x) = m(x)g(x)$. Find the code polynomial for the same $m(x)$ when using systematic code.