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Link Prediction in Graph Streams

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Graph Streams

- Graph Streams arise in a wide variety of applications
  - Activity-centric social and information networks
  - Communication networks
  - Chat networks

- Graph streams are ubiquitous in settings in which activity is overlaid on network structure

Challenges

- Difficult to store the entire graph on disk because of high volume stream

- Graph applications such as link prediction require structural understanding of the graph

- Key is to design probabilistic summaries that can work in the link-prediction setting

The Link Prediction Problem

- Given a network $G = (N, A)$, determine the most likely pairs of nodes that receive a link between them.

- Typical classes of algorithms:
  - Neighborhood-based
  - Matrix factorization
  - Supervised

Contributions of this Work

- Link prediction algorithm for graph streams

- Design summarization schemes to use small-space data structures for prediction

- Is able to achieve results closer to their exact counterparts

- Design the real-time analogs to Jaccard, Adamic-Adar, and common neighbor.

Key Techniques Used

- We design MinHash based graph sketches to estimate Jaccard coefficient.

- Vertex-biased reservoir sampling based graph sketches to estimate common neighbor and Adamic-Adar.
  - Provide theoretical guarantees to exact estimation
  - Robust performance with respect to exact estimation

Streaming Setting

- We consider the streaming link prediction problem in a graph stream that receives a sequence $(e_0, e_1, \ldots, e_t, \ldots)$ of edges.
  - Each of which is in the form of $e_i = (u, v)$ at the time point i, where $u, v \in V$ are incident vertices of the edge e_i.
- We assume the underlying graph is *undirected*, where the ordering of u and v of the edge is insignificant.
- The proposed graph sketches and corresponding estimation algorithms can be generalized to *directed* graphs with minor revision.

Formalization

- At any given moment of time t, the edges seen thus far from the graph stream imply a conceptual graph $G(t) = (V(t), E(t))$, where $V(t)$ is the set of vertices, and $E(t)$ is the set of *distinct* edges up to time t.

- We use $\tau(u,t)$ to represent the set of adjacent vertices of the vertex u in the graph $G(t)$.

  – In other words, $\tau(u,t)$ contains the *distinct* vertices adjacent to u in the graph stream till t, and the degree of u is denoted as $d(u,t) = |\tau(u,t)|$.

- Given a graph stream $G(t)$ at time t, the streaming link prediction problem is to predict whether there is or will be an edge $e = (u,v)$ for any pair of vertices $u, v \in V(t)$ and $e \notin E(t)$.

Target Measures

- In a graph stream $G(t)$ and for any $u, v \in V(t)$, the target measures for streaming link prediction are defined as follows,

  1. **Preferential attachment:** $|\tau(u,t)| \times |\tau(v,t)|$;

  2. **Common neighbor:** $|\tau(u,t) \cap \tau(v,t)|$;

  3. **Jaccard coefficient:** $\frac{|\tau(u,t) \cap \tau(v,t)|}{|\tau(u,t) \cup \tau(v,t)|}$;

  4. **Adamic-Adar:** $\sum_{w \in \tau(u,t) \cap \tau(v,t)} \frac{1}{\log(|\tau(w,t)|)}$

Observations

- These measures seem to be relatively trivial to compute in the static setting.

- However, in the streaming setting, we *do not have a global view of the graph* at any given time.

  - This makes simple computations surprisingly difficult

- As an example, let us look at the simplest measure of preferential attachment

Preferential Attachment

- Preferential attachment, it requires an accurate estimation of the number of distinct edges incident on *each vertex*, *i.e.*, $|\tau(u,t)|, u \in V(t)$.
  - This is not as easy as it sounds because the distinct edges cannot be explicitly maintained and updated for exact counting in a graph stream.

- Streaming methods for distinct element counting may be employed.

- Other neighborhood methods are much harder and will be the focus of the paper.

Jaccard Coefficient

- Natural approach is the min-hash index: has been used earlier in various applications for computation of Jaccard coefficient (Broder et al)

- Given a graph stream, we maintain a MinHash based graph sketch for each vertex u with two key values: the *minimum adjacent hash value*, $H(u)$, and the *minimum adjacent hash index*, $I(u)$.

$$H(u) = min\mathcal{H}(v), (u, v) \in E(t) \tag{1}$$

$$I(u) = argmin\mathcal{H}(v), (u, v) \in E(t) \tag{2}$$

- The Jaccard coefficient between a pair of nodes is equal to the probability that their min-hash indices are the same

Basic Intuition

- Consider two columns in a binary matrix
- if you sort the rows randomly, what is the probability that both columns show values of 1, when at least one of them shows values of 1
  - Simulated by the Jaccard coefficient
  - Also captured by the min-hash index: Each hash function simulates a sort and multiple hash functions are used for robustness
  - Can use Chernoff bound to provide guarantee

Common Neighbor Estimation

- To estimate common neighbors of two vertices in a graph stream, a natural question arises: *whether a sample of edges from the graph stream are good enough to estimate this target measure accurately?*
  - Unfortunately, unbiased sampling provides a negative answer to this question mainly due to the power-law degree distribution of real-world graphs.

Example

- For example, consider a graph in which the top 1% of the high-degree vertices (degrees greater than 10) contain 99% of the edges.

- A down-sampling with a rate of 10% will capture the neighborhood information of such top 1% high-degree vertices robustly, but may not capture even a single edge for the remaining 99% low-degree vertices.
  - Result: poor predictions!

Solution: Vertex-biased Sampling

- To address this issue, we design the *vertex-biased* sampling based graph sketches, where for each vertex $u \in G(t)$, a **reservoir** $S(u)$ of budget L is associated to dynamically sample L incident edges of u.

- Note if an edge (u, v) is sampled, the vertex v, not the edge itself, is maintained and updated in u's reservoir, $S(u)$).

- This sampling approach is biased, because the number of sampled edges, L, is fixed for all vertices in the graph stream, and is therefore disproportionately higher for low-degree vertices than high-degree ones.

Issues

- Sampling L incident edges of each vertex can be implemented by the traditional reservoir sampling method.

- Another problem arises for high-degree vertices instead: for two vertices with degrees larger than L, the number of common neighbors is hard to estimate accurately from their *independent* samples.

- For example, consider the case where the budget is $L = 10$, and the two vertices u and v have a degree $1,000$ each with all such $1,000$ incident vertices being common neighbors of u and v.

- Then, if 10 adjacent neighbors of u and v are sampled independently, the expected number of common neighbors of

u and v is $10 * 10/1000 = 0.1$, which definitely is not an accurate estimation.

Solutions

- To this end, we adopt this constant budget-based sampling approach with an important caveat: *random samples in different reservoirs of vertices are forced to be* **dependent** *on each other*.

- To model this dependency, we consider an implicit sorting order of vertices to impose a priority order on the vertex set of the graph stream.

- Such a priority order is enforced with the use of a hash function $\mathcal{G} : u \in V \to (0, 1)$, where the argument u is a vertex identifier, and the output is a real number in the range $(0, 1)$ with lower values indicating higher priority.

- Specifically, vertices retained in the reservoir $S(u)$ are those having the highest L priority orders among all incident neighbors of u.

Sampling Rate

- In order to dynamically maintain and update $S(u)$ when the graph stream evolves, it is important to note that, if the degree of the vertex u satisfies $d(u,t) > L$, only a fraction $\eta(u,t) = L/d(u,t)$ of all the incident vertices of u can be retained in the sketch $S(u)$.

- To account for this, we define a *threshold* of the priority value $\mathcal{G}(\cdot)$ for each incident neighbor of u that can survive in the reservoir $S(u)$ as

$$\eta(u,t) = \min\{1, L/d(u,t)\} \tag{3}$$

Estimating number of common neighbors

- Let $\eta(u,t)$ and $\eta(v,t)$ be the fraction (threshold) of incident neighbors of u and v, respectively, which are sampled. The number of common neighbors, C_{uv}, of u and v is

$$C_{uv} = \frac{|S(u) \cap S(v)|}{\max\{\eta(u,t), \eta(v,t)\}} \tag{4}$$

- Paper discusses the trick to dynamically maintain $\eta(u,t)$ and $\eta(v,t)$.

Adamic-Adar

- An important observation of Adamic-Adar is that the common neighbor w of vertices u and v with a higher vertex-degree is weighted less, because of its proclivity to be a noisy vertex, as accounted for in the term $1/\log(|\tau(w,t)|)$.

- Such a weighting strategy makes it very challenging to estimate Adamic-Adar in graph streams.

- Very high-degree vertices contribute noice.

Truncated Adamic-Adar

- We consider an approximate variation of Adamic-Adar, which *truncates* the insignificant high-degree common neighbors of u and v for streaming link prediction, because they contribute little in the computation of Adamic-Adar.

- It turns out that such an approximation still preserves the accuracy of Adamic-Adar, but can be computed *exactly* based on the vertex-biased sampling based graph sketches.

- The truncated Adamic-Adar, denoted as $\overline{AA}(u, v)$, between two vertices u and v, is defined in the same way as Adamic-Adar, except that the components contributed by common neighbors with degrees greater than d_{max} are eliminated:

$$\overline{AA}(u, v) = \sum_{\{w \in \tau(u,t) \cap \tau(v,t) : |\tau(w,t)| \leq d_{max}\}} \frac{1}{\log(|\tau(w, t)|)} \tag{5}$$

Approach

- The core idea is that each of the components in the equation can be directly computed if both vertices u and v are present in the graph sketch $S(w)$ of their common neighbor w, whose degree is no larger than d_{max}.

- At the beginning, we initialize the capacity of the reservoirs, $L = d_{max}$, for all vertices.

- This is because vertices whose degrees are larger than d_{max} will not be considered in the computation of truncated Adamic-Adar

Truncated Adamic-Adar

- We then examine all the graph sketches one by one. For each vertex w, both vertices u and v need be present in $S(w)$ (Line 5).

- It is further checked whether w has a degree at most d_{max} by examining if the values of $\underline{\eta}(w)$ and $\overline{\eta}(w)$ equal to 1, indicating that the graph sketch $S(w)$ is not full yet .

- If so, (1/log($|\tau(w,t)|$)) is added to $\overline{AA}(u,v)$.

- Note that the degree value, $|\tau(w,t)|$, is equal to the current reservoir size, $|S(w)|$.

Handling computational bottlenecks

- The main computational bottleneck is that the graph sketches of all vertices need to be scanned once for each incoming new edge (u, v) in the graph stream.

- To alleviate this problem, we consider adopting inverted indexes to facilitate the computation of truncated Adamic-Adar.

- Specifically, for each vertex u in the graph stream, an auxiliary inverted index structure, $L(u)$, is built to maintain vertex identifiers of v whose graph sketch $S(v)$ contains u, *i.e.*, $v \in L(u)$ if and only if $u \in S(v)$.

- Inverted indices are dynamically maintained and they double the memory requirement.

Experimental Results

- We chose three real-world, publicly available graph datasets in our experimental studies.

- Note that their edges are attached with timestamps indicating when they are first created in the graphs, and thus can be ordered, formulated, and processed in a form of graph streams
  - DBLP, Amazon Product co-purchasing, Wikipedia citation network

Prediction Accuracy

- Compute accuracy with respect to a random predictor
- Ratio of the accuracy to that of a random predictor
- Values greater than 1 are good.

DBLP Results

Methods	Ext-Ja	App-Ja	Ext-CN	App-CN	Ext-Adar	App-Adar
Accuracy	120.57	104.25	112.66	108.91	93.47	92.57

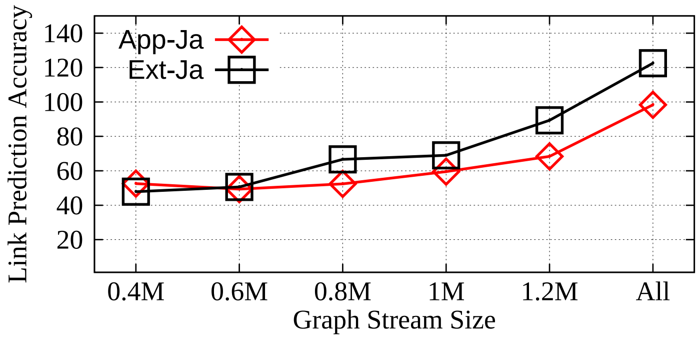
Methods	Katz	PrefAttach	PropFlow	R-PRank	Short-Path	SimRank
Accuracy	100.48	114.97	96.76	59.82	107.49	74.85

Amazon Co-Purchasing Network

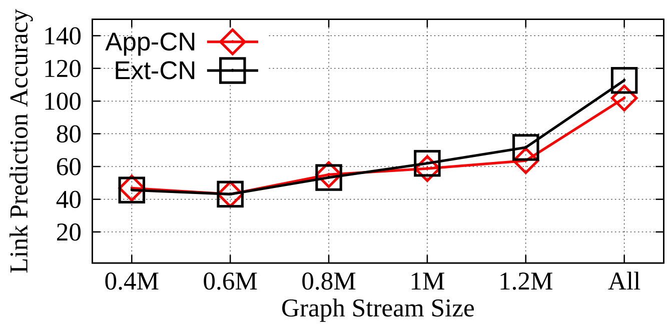
Methods	Ext-Ja	App-Ja	Ext-CN	App-CN	Ext-Adar	App-Adar
Accuracy	116.15	106.74	121.07	109.70	116.57	116.15

Methods	Katz	PrefAttach	PropFlow	R-PRank	Short-Path	SimRank
Accuracy	88.38	116.23	98.75	71.48	147.70	110.95

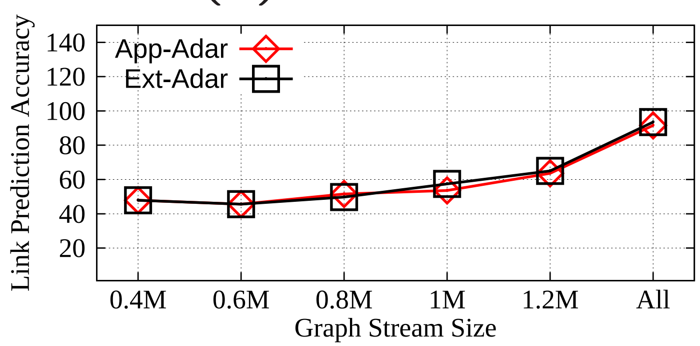
Progression with Stream Size (DBLP)



(a) Jaccard

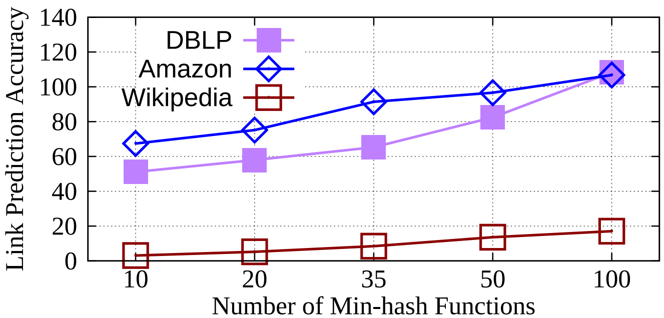


(b) Common neighbor

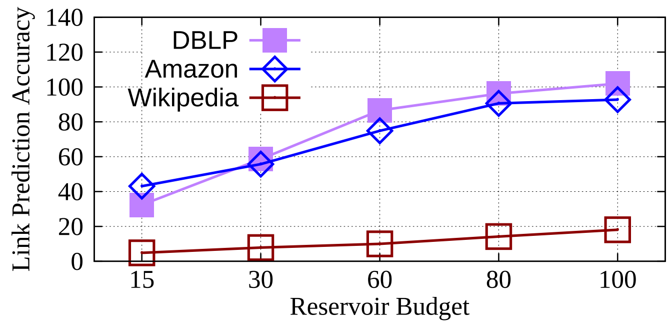


(c) Adamic-Adar

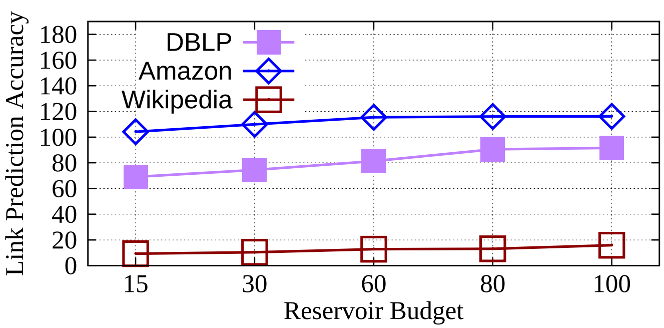
Performance with respect to algorithm parameters



(a) Jaccard

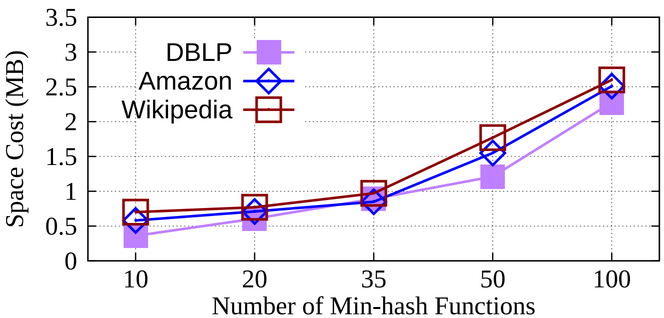


(b) Common Neighbor

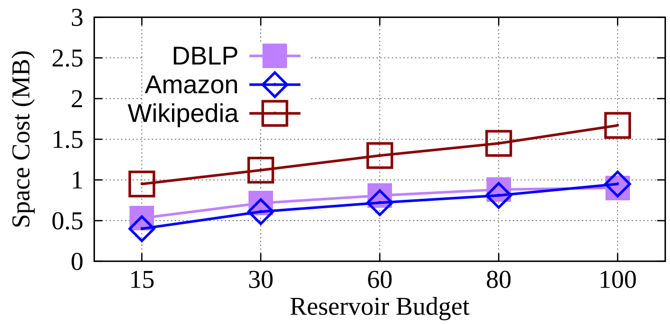


(c) Adamic-Adar

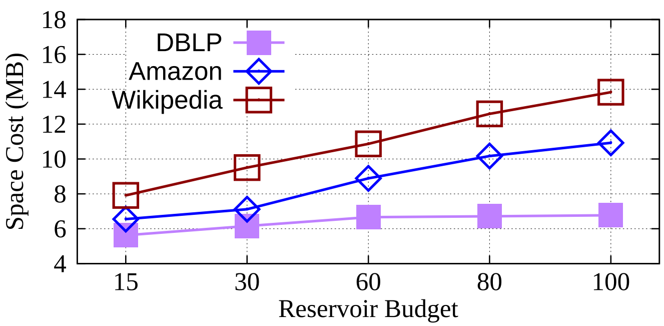
Space with respect to algorithm parameters



(a) Jaccard



(b) Common Neighbor



(c) Adamic-Adar

Runtime Cost vs Graph Stream Size

Runtime Cost (in seconds)	Graph Stream Size S			
	0.4M	0.8M	1.2M	All
App-Ja	0.004	0.04	0.09	0.12
Ext-Ja	21.68	135.16	833.50	2748.40
App-CN	0.007	0.048	0.09	0.395
Ext-CN	18.0	194.31	801.72	2935.22
App-Adar	0.29	4.45	5.05	19.86
Ext-Adar	16.44	260.13	1193.77	3308.26

Conclusions

- New method for link prediction in graph streams

- Generalizes common neighborhood methods for graph streams

- Future work will also generalize more advanced techniques for graph streams

- Design methods for incorporating content in link prediction