# Message Scheduling for All-to-All Personalized Communication on Ethernet Switched Clusters *

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## Abstract

*We develop a message scheduling scheme that can theoretically achieve the maximum throughput for all–to–all personalized communication (AAPC) on any given Ethernet switched cluster. Based on the scheduling scheme, we implement an automatic routine generator that takes the topology information as input and produces a customized MPI_Alltoall routine, a routine in the Message Passing Interface (MPI) standard that realizes AAPC. Experimental results show that the automatically generated routine consistently out-performs other MPI_Alltoall algorithms, including those in LAM/MPI and MPICH, on Ethernet switched clusters with different network topologies when the message size is sufficiently large. This demonstrates the superiority of the proposed AAPC algorithm in exploiting network bandwidths.*

## 1. Introduction

All–to–all personalized communication (AAPC) is one of the most common communication patterns in high performance computing. In AAPC, each node in a system sends a different message of the same size to every other node. The Message Passing Interface routine that realizes AAPC is *MPI_Alltoall* [11]. AAPC appears in many high performance applications, including matrix transpose, multi-dimensional convolution, and data redistribution. Since AAPC is often used to rearrange the whole global array in an application, the message size in AAPC is usually large. Thus, it is crucial to have an AAPC implementation that can fully exploit the network bandwidth in the system.

Switched Ethernet is the most widely used local–area–network technology. Many Ethernet switched clusters of workstations are used to perform high performance computing. For such clusters to be effective, communications must be carried out as efficiently as possible. In this paper, we investigate efficient AAPC on Ethernet switched clusters.

We develop a message scheduling scheme that theoretically achieves the maximum throughput of AAPC on any given Ethernet switched cluster. Similar to other AAPC scheduling schemes [5], our scheme partitions AAPC into contention free phases. It achieves the maximum throughput by fully utilizing the bandwidth in the bottleneck links in all phases. Based on the scheduling scheme, we develop an automatic routine generator that takes the topology information as input and produces an *MPI_Alltoall* routine that is customized for the specific topology. We compare the automatically generated routine with the original routine in LAM/MPI [8] and a recently improved *MPI_Alltoall* implementation in MPICH [18]. The results show that the automatically generated routine consistently outperforms the existing algorithms when the message size is sufficiently large, which demonstrates the superiority of the proposed AAPC algorithm in exploiting network bandwidths.

The rest of the paper is organized as follows. Section 2 discusses the related work. Section 3 describes the network model and defines the scheduling problem. Section 4 details the proposed scheduling scheme. Section 5 discusses implementation issues. Section 6 reports experimental results. Finally, the conclusions are presented in Section 7.

## 2. Related Work

AAPC has been extensively studied due to its importance. A large number of optimal message scheduling algorithms for different network topologies with different network models were developed. Many of the algorithms were designed for specific network topologies that are used in the parallel machines, including hy-

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percube [6, 19], mesh [1, 14, 13, 17], torus [5, 9], k-ary n-cube [19], and fat tree [3, 12]. Heuristic algorithms were developed for AAPC on irregular topologies [10]. A framework for AAPC that is realized with indirect communications was reported in [7]. Efficient AAPC scheduling schemes for clusters connected by a single switch was proposed in [15]. Some of the algorithms in [15] are incorporated in the recent improvement of MPICH library [18]. We consider Ethernet switched clusters with one or more switches. AAPC on such clusters is a special communication pattern on a tree topology. To the best of our knowledge, message scheduling for such cases has not been developed.

## 3. Network Model and Problem Definition

In an Ethernet switched network, machines connected to an Ethernet switch can send and receive at the full link speed simultaneously since links operate in the duplex mode. The switches use a spanning tree algorithm to determine forwarding paths that follow a tree structure [16]. Thus, the physical topology of the network is always a **tree**.

The network can be modeled as a directed graph $G = (V, E)$ with nodes $V$ corresponding to switches and machines, and edges $E$ corresponding to unidirectional channels. Let $S$ be the set of all switches in the network and $M$ be the set of all machines in the network ($V = S \cup M$). Let $u, v \in V$, a directed **edge** $(u, v) \in E$ if and only if there is a link between node $u$ and node $v$. We will call the physical connection between node $u$ and node $v$ **link** $(u, v)$, which corresponds to two directed edges $(u, v)$ and $(v, u)$. Since the network topology is a tree, the graph is also a tree: there is a unique path between any two nodes. Node $u \in M$ can only be a leaf node. Figure 1 shows an example cluster.

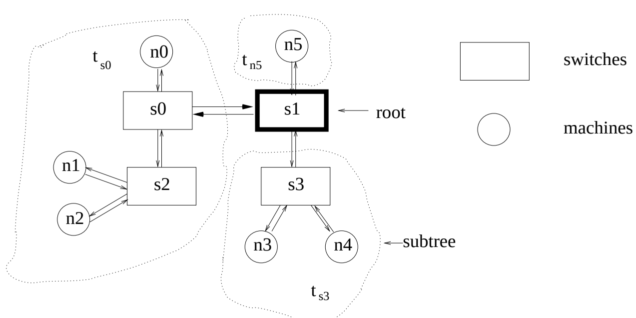



**Figure 1. An example Ethernet Switched Cluster**

The terminologies used in this paper are defined next. A *message*, $u \to v$, is a communication transmitted from node $u$ to node $v$. The notion $path(u, v)$ denotes the set of directed edges in the unique path from node $u$ to node $v$. For example, in Figure 1, $path(n0, n3) = \{(n0, s0), (s0, s1), (s1, s3), (s3, n3)\}$. Two messages, $u_1 \to v_1$ and $u_2 \to v_2$, are said to have contention if they share a common edge. A *pattern* is a set of messages. The *AAPC pattern* on a network $G = (S \cup M, E)$ is $\{u \to v \mid u \neq v;\ u, v \in M\}$. The message size is denoted as *msize*. In a *contention free pattern*, no two messages have contention. A *phase* is a contention free pattern. The *load* on an edge is the number of times the edge is used in the pattern. The most loaded edge is called a *bottleneck* edge. Since the topology is a tree, edges $(u, v)$ and $(v, u)$ always have the same load. We will use the terms "the load of an edge $(u, v)$" and "the load of a link $(u, v)$" interchangeably. The *load of AAPC pattern* is equal to the load of a bottleneck edge. $|S|$ denotes the size of a set $S$. Since scheduling for AAPC when $|M| \leq 2$ is trivial, we will assume that $|M| \geq 3$.

Let edge $(u, v)$ be one of the bottleneck edges for the AAPC pattern. Assume that removing link $(u, v)$ partitions graph $G = (S \cup M, E)$ into two sub-graphs, $G_u = (S_u \cup M_u, E_u)$ and $G_v = (S_v \cup M_v, E_v)$. $G_u$ is the connected component including node $u$, and $G_v$ is the connected component including node $v$. AAPC requires $|M_u| \times |M_v| \times msize$ bytes data to be transferred across the link $(u, v)$ in both directions. Let $B$ be the bandwidth on all links. The best case time to complete AAPC is $\frac{|M_u| \times |M_v| \times msize}{B}$. The peak aggregate throughput of AAPC is bounded by

$$\frac{|M| \times (|M| - 1) \times \ msize}{\frac{|M_u| \times |M_v| \times msize}{B}} = \frac{|M| \times (|M| - 1) \times \ B}{M_u \times M_v}$$

In general networks, this peak aggregate throughput may not be achieved due to node and link congestion. However, as will be shown later, for the tree topology, this physical limit can be approached through message scheduling.

## 4. AAPC Message Scheduling

In the following, we will present an algorithm that constructs phases for AAPC. The phases conform to the following constraints, which are sufficient to guarantee optimality: (1) no contention within each phase; (2) every message in AAPC appears exactly once in the phases; and (3) the total number of phases is equal to the load of AAPC on a given topology. If phases that satisfy these constraints can be carried out without inter-phase interferences, the peak aggregate throughput is achieved.

Our scheduling algorithm has three components. The first component identifies the *root* of the system. For a graph $G = (S \cup M, E)$, the *root* is a switch that satisfies two conditions: (1) it is connected to a bottleneck edge; and (2) the number of machines in each of the subtrees connecting to the root is less than or equal to $\frac{|M|}{2}$. The second component performs global message scheduling that determines the phases when messages between two subtrees are carried out. Finally, the third component performs global and local message assignment, which decides the final scheduling of local and global messages.

## 4.1. Identifying the Root

Let the graph be $G = (S \cup M, E)$. The process to find a root in the network is as follows. Let link $L = (u, v)$ be one of the bottleneck links. Link $L$ partitions the graph into two subgraphs $G_u$ and $G_v$. The load of link $L$ is thus, $|M_u| \times |M_v| = (|M| - M_v) \times |M_v|$. Assume that $|M_u| \geq |M_v|$. If in $G_u$, node $u$ has more than one branch containing machines, then node $u$ is the root. Otherwise, node $u$ should have exactly one branch that contains machines (obviously this branch may also have switches). Let the branch connect to node $u$ through link $(u_1, u)$. Clearly, link $(u_1, u)$ is also a bottleneck link since all machines in $G_u$ are in $G_{u_1}$. Thus, we can repeat the process for link $(u_1, u)$. This process can be repeated $n$ times and $n$ bottleneck links $(u_n, u_{n-1})$, $(u_{n-1}, u_{n-2})$, ..., $(u_1, u)$, are considered until the node $u_n$ has more than one branch containing machines in $G_{u_n}$. Then, $u_n$ is the root. Node $u_n$ should have a nodal degree larger than 2 in $G$.

**Lemma 1**: Using the above process to find the root, each subtree of the root contains at most $\frac{|M|}{2}$ machines.
*Proof*: See [4].

## 4.2. Global Message Scheduling

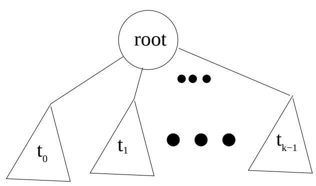



**Figure 2. A two level view of the network**

Let the root connect to $k$ subtrees, $t_0$, $t_1$, ..., and $t_{k-1}$, with $|M_0|$, $|M_1|$, ..., and $|M_{k-1}|$ machines respectively. Figure 2 shows the two-level view of the network. Only global messages use the links between the subtrees and the root. Local messages only use links within a subtree. Let us assume that $|M_0| \geq |M_1| \geq ... \geq |M_{k-1}|$. Thus, the load of AAPC is $|M_0| \times (|M_1| + |M_2| + ... + |M_{k-1}|) = |M_0| \times (|M| - |M_0|)$, and we must schedule both local and global messages in $|M_0| \times (|M| - |M_0|)$ phases while maintaining contention-free phases. This is done in two steps. First, phases are allocated for global messages where messages from one subtree to another subtree are treated as groups. Second, individual global and local messages are assigned to particular phases.

**Table 1. Phases from ring scheduling**

| Phase 0 | Phase 1 | ... | Phase $k-2$ |
|---|---|---|---|
| $t_0 \rightarrow t_1$ | $t_0 \rightarrow t_2$ | ... | $t_0 \rightarrow t_{k-1}$ |
| $t_1 \rightarrow t_2$ | $t_1 \rightarrow t_3$ | ... | $t_1 \rightarrow t_0$ |
| ... | ... | ... | ... |
| $t_{k-2} \rightarrow t_{k-1}$ | $t_{k-2} \rightarrow t_0$ | ... | $t_{k-2} \rightarrow t_{k-3}$ |
| $t_{k-1} \rightarrow t_0$ | $t_{k-1} \rightarrow t_1$ | ... | $t_{k-1} \rightarrow t_{k-2}$ |

We will use the notation $t_i \rightarrow t_j$ to represent either a message from a machine in subtree $t_i$ to a machine in subtree $t_j$ or general messages from subtree $t_i$ to subtree $t_j$. The global message scheduling decides phases for messages in $t_i \rightarrow t_j$. Let us first consider a simple case where $|M_0| = |M_1| = ... = |M_{k-1}| = 1$. In this case, there is $|M_i| \times |M_j| = 1$ message in $t_i \rightarrow t_j$. A ring scheduling algorithm [18, 15] can be used to schedule the messages in $1 \times (k-1) = k-1$ phases. In the ring scheduling, $t_i \rightarrow t_j$ is scheduled at phase $j - i - 1$ if $j > i$ and phase $(k-1) - (i-j)$ if $i > j$. The ring scheduling produces $k-1$ phases as shown in Table 1.

When scheduling messages with any number of machines in a subtree, we group all messages from one subtree to another into consecutive phases. The total number of messages from $t_i$ to $t_j$ is $|M_i| \times |M_j|$. We extend ring scheduling to allocate phases for groups of messages. In the extended ring scheduling, for subtree $t_i$, the messages to other subtrees follow the same order as the ring scheduling. For example, for $t_1$, messages in $t_1 \rightarrow t_2$ happen before messages in $t_1 \rightarrow t_3$, messages in $t_1 \rightarrow t_3$ happen before messages in $t_1 \rightarrow t_4$, and so on. Specifically, the phases are allocated as follows. Note that messages in $t_i \rightarrow t_j$ occupy $|M_i| \times |M_j|$ consecutive phases.

- When $j > i$, messages in $t_i \rightarrow t_j$ start at phase $p = |M_i| \times \sum_{k=i+1}^{j-1} |M_k|$. If $i + 1 > j - 1$, $p = 0$.
- When $i > j$, messages in $t_i \rightarrow t_j$ start at phase $p = |M_0| \times (|M| - |M_0|) - (|M_j| \times \sum_{k=j+1}^{i} |M_k|)$.

**Lemma 2**: Using the extended ring scheduling described above, the resulting phases have the following two properties: (1) the number of phases is $|M_0| \times (|M| - |M_0|)$; and (2) in each phase, global messages do not have contention on links connecting subtrees to the root.
*Proof*: See [4].

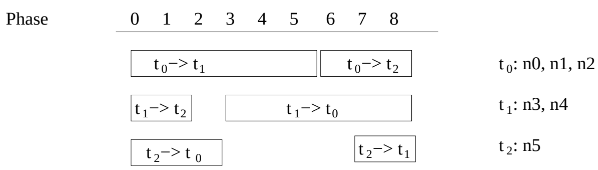



**Figure 3. Global message scheduling for the example in Figure 1**

Figure 3 shows the scheduling of global messages for the example shown in Figure 1. In this figure, there are three subtrees: $t_0 = t_{s0}$ with $|M_0| = 3$, $t_1 = t_{s3}$ with $|M_1| = 2$, and $t_2 = t_{n5}$ with $|M_2| = 1$. Following the above equations, messages in $t_1 \rightarrow t_2$ start at $p = 0$, messages in $t_0 \rightarrow t_2$ start at $p = 6$, and messages in $t_2 \rightarrow t_0$ start at $p = 0$. Figure 3 also shows that some subtrees are idle at some phases. For example, subtree $t_1$ does not have a sending machine in phase 2.

### 4.3. Global and Local Message Assignment

Let the root connect to $k$ subtrees, $t_0$, $t_1$, ..., $t_{k-1}$, with $|M_0|$, $|M_1|$, ..., $|M_{k-1}|$ machines respectively. $|M_0| \geq |M_1| \geq ... \geq |M_{k-1}|$. As shown in the previous subsection, global messages are scheduled in $|M_0| \times (|M| - |M_0|)$ phases. Consider subtree $t_i$, the total number of local messages in $t_i$ is $|M_i| \times (|M_i| - 1)$, which is less than the total number of phases. Thus, if in each phase, one local message in each subtree can be scheduled without contention with the global messages, all messages in AAPC can be scheduled in $|M_0| \times (|M| - |M_0|)$ phases. The contention free scheduling of global and local messages is based on the following lemma.

**Lemma 3**: Let $G = (S \cup M, E)$ be a tree and $x \neq y \neq z \in S \cup M$, $path(x, y) \cap path(y, z) = \phi$.

*Proof*: Assume that $path(x, y) \cap path(y, z) \neq \phi$. There exists an edge $(u, v)$ that belongs to both $path(x, y)$ and $path(y, z)$. As a result, the composition of the partial path $path(y, u) \subseteq path(y, z)$ and $path(u, y) \subseteq path(x, y)$ forms a non-trivial loop: edge $(u, v)$ is in the loop while edge $(v, u)$ is not. This contradicts the assumption that $G$ is a tree. □

**Lemma 4**: Using the global message scheduling scheme, at each phase, the global messages do not have contention.

*Proof:* Let the root connect to subtrees $t_0$, $t_1$, ..., $t_{k-1}$. From Lemma 2, at each phase, there is no contention in the link connecting a subtree to the root. Also, there is no contention when there is only one global message in a subtree in a phase. Thus, the only case when global messages may have contention inside a subtree is when there are two global messages involving nodes in a subtree in a phase: one global message, $x \rightarrow o_1$, is sent into the subtree and the other one, $o_2 \rightarrow y$, is sent out from the subtree ($x, y \in M_i$; $o_1$ and $o_2$ are in other subtrees). The sub-path for $x \rightarrow o_1$ inside $t_i$ is equal to $path(x, root)$ and the sub-path for $o_2 \rightarrow y$ is equal to $path(root, y)$. From Lemma 3, these two paths do not have contention inside $t_i$. □

The contention free scheduling of local messages is also based on Lemma 3. Let $u, v \in t_i$ *and* $u \neq v$. From Lemma 3, there are three cases when message $u \rightarrow v$ can be scheduled without contention (with global messages) in a phase: (1) node $v$ is the sender of a global message, and node $u$ is the receiver of a global message; (2) node $v$ is the sender of a global message, and there is no receiving node of a global message in $t_i$; and (3) node $u$ is the receiver of a global message, and there is no sending node of a global message. Note that by scheduling at most one local message in each subtree, our scheduling algorithm does not have to consider the specific topologies of the subtrees.

Let us now consider global messages assignment. Let us number the nodes in subtree $t_i$ as $t_{i,0}$, $t_{i,1}$, ..., $t_{i,(|M_i|-1)}$. To realize inter-subtree communication $t_i \rightarrow t_j$, $0 \leq i \neq j < k$, each node in $t_i$ must communicate with each node in $t_j$ in the allocated $|M_i| \times |M_j|$ phases. The algorithm uses two different methods to realize inter-subtree communications. The first scheme is the *broadcast* scheme. In this scheme, the $|M_i| \times |M_j|$ phases are partitioned into $|M_i|$ rounds with each round having $|M_j|$ phases. In each round, a different node in $t_i$ sends one message to each of the nodes in $t_j$. This method has the flexibility in selecting the order of the senders in $t_i$ in each round and the order of the receivers in $t_j$ within each round. The following pattern is an example of such scheme:
$t_{i,0} \rightarrow t_{j,0}, ..., t_{i,0} \rightarrow t_{j,|M_j|-1}, t_{i,1} \rightarrow t_{j,0}, ..., t_{i,1} \rightarrow t_{j,|M_j|-1}$, ..., $t_{i,|M_i|-1} \rightarrow t_{j,0}, ..., t_{i,|M_i|-1} \rightarrow t_{j,|M_j|-1}$.

The second scheme is the *rotate* scheme. Let $D$ be the greatest common divisor of $|M_i|$ and $|M_j|$. Thus, $|M_i| = a \times D$ and $|M_j| = b \times D$. In this scheme, the pattern for receivers is a repetition of $M_i$ times of some fixed sequence that enumerates all nodes in $t_j$. One example of a fixed sequence is $t_{j,0}, t_{j,1}, ...t_{j,|M_j|-1}$, which results in a receiver pattern of $t_{j,0}, t_{j,1}, ...t_{j,|M_j|-1}, t_{j,0}, t_{j,1}, ...t_{j,|M_j|-1}, ..., t_{j,0}, t_{j,1}, ... t_{j,|M_j|-1}$. Unlike the broadcast scheme, in a rotate scheme, the sender pattern is also an enumeration of all nodes in $t_i$ in every $|M_i|$ phases. There is a *base sequence* for the senders, which can be an arbitrary sequence that covers all nodes in $t_i$. In the scheduling, the base sequence and the "rotated" base sequence are used. Let the base sequence be $t_{i,0}, t_{i,1}, ...t_{i,|M_i|-1}$. The base sequence

**Table 2. Rotate pattern for realizing $t_i \rightarrow t_j$ when $|M_i| = 6$ and $|M_j| = 4$**

| phase | comm. | phase | comm | phase | comm | phase | comm |
|---|---|---|---|---|---|---|---|
| 0 | $t_{i,0} \rightarrow t_{j,0}$ | 6 | $t_{i,0} \rightarrow t_{j,2}$ | 12 | $t_{i,1} \rightarrow t_{j,0}$ | 18 | $t_{i,1} \rightarrow t_{j,2}$ |
| 1 | $t_{i,1} \rightarrow t_{j,1}$ | 7 | $t_{i,1} \rightarrow t_{j,3}$ | 13 | $t_{i,2} \rightarrow t_{j,1}$ | 19 | $t_{i,2} \rightarrow t_{j,3}$ |
| 2 | $t_{i,2} \rightarrow t_{j,2}$ | 8 | $t_{i,2} \rightarrow t_{j,0}$ | 14 | $t_{i,3} \rightarrow t_{j,2}$ | 20 | $t_{i,3} \rightarrow t_{j,0}$ |
| 3 | $t_{i,3} \rightarrow t_{j,3}$ | 9 | $t_{i,3} \rightarrow t_{j,1}$ | 15 | $t_{i,4} \rightarrow t_{j,3}$ | 21 | $t_{i,4} \rightarrow t_{j,1}$ |
| 4 | $t_{i,4} \rightarrow t_{j,0}$ | 10 | $t_{i,4} \rightarrow t_{j,2}$ | 16 | $t_{i,5} \rightarrow t_{j,0}$ | 22 | $t_{i,5} \rightarrow t_{j,2}$ |
| 5 | $t_{i,5} \rightarrow t_{j,1}$ | 11 | $t_{i,5} \rightarrow t_{j,3}$ | 17 | $t_{i,0} \rightarrow t_{j,1}$ | 23 | $t_{i,0} \rightarrow t_{j,3}$ |

**Table 3. Mapping between senders and the receivers in Step 2.**

| round 0 | | round 1 | | ... | round $\|M_0\|-2$ | | round $\|M_0\|-1$ | | ... |
|---|---|---|---|---|---|---|---|---|---|
| send | recv | send | recv | ... | send | recv | send | recv | ... |
| $t_{0,0}$ | $t_{0,1}$ | $t_{0,0}$ | $t_{0,2}$ | ... | $t_{0,0}$ | $t_{0,\|M_0\|-1}$ | $t_{0,0}$ | $t_{0,0}$ | ... |
| $t_{0,1}$ | $t_{0,2}$ | $t_{0,1}$ | $t_{0,3}$ | ... | $t_{0,1}$ | $t_{0,0}$ | $t_{0,1}$ | $t_{0,1}$ | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| $t_{0,\|M_0\|-2}$ | $t_{0,\|M_0\|-1}$ | $t_{0,\|M_0\|-2}$ | $t_{0,0}$ | ... | $t_{0,\|M_0\|-2}$ | $t_{0,\|M_0\|-3}$ | $t_{0,\|M_0\|-2}$ | $t_{0,\|M_0\|-2}$ | ... |
| $t_{0,\|M_0\|-1}$ | $t_{0,0}$ | $t_{0,\|M_0\|-1}$ | $t_{0,1}$ | ... | $t_{0,\|M_0\|-1}$ | $t_{0,\|M_0\|-2}$ | $t_{0,\|M_0\|-1}$ | $t_{0,\|M_0\|-1}$ | ... |

can be rotated once, which produces the sequence $t_{i,1}, ...t_{i,|M_i|-1}, t_{i,0}$. Sequence $t_{i,2}, ...t_{i,|M_i|-1}, t_{i,0}, t_{i,1}$ is the result of rotating the base sequence twice. The result from rotating the base sequence $n$ times can be defined similarly. The senders are scheduled as follows. The base sequence is repeated $b$ times for the first $a \times b \times D$ phases. Then, at phases that are multiples of $a \times b \times D$ phases, rotations are performed to find a new sequence that is repeated $b$ times. It can be shown that all messages in $t_i \rightarrow t_j$ are realized in the rotate scheme.

Table 2 shows an example when $|M_i| = 6$ and $|M_j| = 4$. In this case, $a = 3$, $b = 2$, and $D = 2$. The receivers repeat the pattern $t_{j,0}, t_{j,1}, t_{j,2}, t_{j,3}$. The base sequence for the senders is $t_{i,0}, t_{i,1}, t_{i,2}, t_{i,3}, t_{i,4}, t_{i,5}$. This sequence is repeated 2 times. At phase $2 * 3 * 2 = 12$, the senders follow a rotated sequence $t_{i,1}, t_{i,2}, t_{i,3}, t_{i,4}, t_{i,5}, t_{i,0}$ and repeat the pattern 2 times. It can be verified that all messages in $t_i \rightarrow t_j$ are realized.

The following two lemmas illustrate the properties of the broadcast pattern and the rotate pattern.

**Lemma 5**: In the broadcast pattern that realizes $t_i \rightarrow t_j$, each sender $t_{i,k}$ occupies $|M_j|$ continuous phases.

*Proof*: Straight-forward from the definition of the broadcast pattern. □.

**Lemma 6**: In the rotate pattern that realizes $t_i \rightarrow t_j$, counting from the first phase for messages in $t_i \rightarrow t_j$, each sender in $t_i$ happens once in every $|M_i|$ phases and each receiver in $t_j$ happens once in every $|M_j|$ phases.

*Proof*: Straight-forward from the definition of the rotate pattern. □.

Either the broadcast pattern or the rotate pattern can be used to realize messages in $t_i \rightarrow t_j$, $0 \leq i \neq j < k$. The challenge in the scheduling, however, is that we must be able to embed all local messages in the $|M_0| \times (|M| - |M_0|)$ phases. The scheduling algorithm is shown in Figure 4. The algorithm consists of six steps. We will explain each step next.

In the first step, messages from $t_0$ to all other subtrees $t_j$, $1 \leq j < k$ are scheduled using the rotate scheme. First, the receivers in $t_0 \rightarrow t_j$ are assigned such that at phase $p$, node $t_{j,(p-|M_0|\times(|M|-|M_0|))\ mod\ |M_j|}$ is the receiver. The pattern aligns the receivers with the receivers in $t_i \rightarrow t_j$ when $i > j$. As will be shown in Step 5, this alignment is needed to correctly schedule local messages. The rotate pattern ensures that each node in $t_0$ appears once as the sender in every $|M_0|$ phases counting from phase 0.

In the second step, messages in $t_i \rightarrow t_0$ are assigned. In this step, phases are partitioned into rounds with each round having $|M_0|$ phases. The main objective of this step is to schedule all local messages in $t_0$. This is achieved by creating the pattern shown in Table 3, which is basically a rotate pattern for $t_0 \rightarrow t_0$. Since in step 1, each node in $t_0$ appears as a sender in every $|M_0|$ phases, the scheduling of receivers in $t_i \rightarrow t_0$ can directly follow the mapping in Table 3. Using this mapping, every node in $t_0$ appears as a receiver in every $|M_0|$ phases, which facilitates the use of a broadcast pattern to realize messages in $t_i \rightarrow t_0$, $i > 0$. After the receiver pattern is decided, the senders of $t_i \rightarrow t_0$ are determined using the broadcast scheme with the sender order $t_{i,0}$, $t_{i,1}$, ..., $t_{i,|M_i|-1}$.

Step 3 embeds local messages in $t_0$ in the first $|M_0| \times (|M_0| - 1)$ phases. Note that $|M_0| \times (|M_0| - 1) \leq |M_0| \times (|M| - |M_0|)$. Since the global messages for nodes in $t_0$ are scheduled according to Table 3, for each $t_{0,n} \rightarrow t_{0,m}$, $0 \leq n \neq m < |M_0|$, there exists a phase in the first $|M_0| \times (|M_0| - 1)$ phases such that $t_{0,n}$ is a receiver of a global message while $t_{0,m}$ is a sender of a global message. Thus, all local messages in $t_0$, $t_{0,n} \rightarrow t_{0,m}$, $0 \leq n \neq m < |M_0|$, can be scheduled in the the first $|M_0| \times (|M_0| - 1)$ phases.

**Input**: Results from global message scheduling that identify which phases are used to realize $t_i \rightarrow t_j$ for all $0 \leq i \neq j < k$

**Output**: (1) the phase to realize each global message $t_{i,i_1} \rightarrow t_{j,j_1}, 0 \leq i_1 < |M_i|, 0 \leq j_1 < |M_j|, 0 \leq i \neq j < k$.

(2) the phase to realize each local message $t_{i,i_1} \rightarrow t_{i,i_2}, 0 \leq i_1 \neq i_2 < |M_i|, 0 \leq i < k$.

Step 1: Assign phases to messages in $t_0 \rightarrow t_j$, $1 \leq j < k$.

1.a: For each $t_0 \rightarrow t_j$, the receivers in $t_j$ are assigned as follows:
at phase $p$ in the phases for $t_0 \rightarrow t_j$, machine $t_{j,(p-|M_0|\times(|M|-|M_0|)) \ mod \ |M_j|}$ is the receiver.
/* it can be verified that a sequence that enumerates the nodes in $t_j$ is repeated $|M_0|$ times in phases for $t_0 \rightarrow t_j$. */

1.b: For each $t_0 \rightarrow t_j$, the senders in $t_0$ are assigned according to the rotate pattern with the base sequence $t_{0,0}, t_{0,1}, ..., t_{0,|M_0|-1}$.

Step 2: Assign phases to messages in $t_i \rightarrow t_0$, $1 \leq i < k$.

2.a: Assign the receivers in $t_i \rightarrow t_0$:
/*Step 1.b organizes the senders in $t_0$ in such a way that every $|M_0|$ phases, all nodes in $t_0$ appear as the sender once. We call $|M_0|$ phases a *round* */
The receiver pattern in $t_i \rightarrow t_0$ is computed based on the sender pattern in $t_0 \rightarrow t_j$ according to the mapping shown in Table 3. Round $r$ has the same mapping as round $r \ mod \ |M_0|$.
/* the mapping ensures that the local messages in $t_0$ can be scheduled */

2.b: Assign the senders in $t_i$ using the broadcast pattern with order $t_{i,0}, t_{i,1}, ..., t_{i,|M_i|-1}$.

Step 3: Schedule local messages in $t_0$ in phase 0 to phase $|M_0| \times (|M_0| - 1)$.
message $t_{0,i} \rightarrow t_{0,j}$, $0 \leq i \neq j < |M_0|$, is scheduled at the phase where $t_{0,i}$ is the receiver of a global message and $t_{0,j}$ is the sender of a global message.

Step 4: Assign phases to global messages in $t_i \rightarrow t_j$, $i > j$ and $j \neq 0$.
Use the broadcast pattern with receivers repeating pattern $t_{j,0}, t_{j,1}, ..., t_{j,|M_j|-1}$ for each sender $t_{i,k}$ and senders following the order $t_{i,0}, t_{i,1}, t_{i,k}, ..., t_{i,|M_i|-1}$.

Step 5: Schedule local messages in $t_i$, $1 \leq i < k$ in phases for $t_i \rightarrow t_{i-1}$.
/* the last phase for $t_i \rightarrow t_{i-1}$ is the last phase $|M_0| \times (|M| - |M_0|) - 1$.*/
Steps 1 through 4 ensure that for each local message $t_{i,i1} \rightarrow t_{i,i2}$, there is a phase in the phases for $t_i \rightarrow t_{i-1}$ such that $t_{i,i2}$ is the sender of a global message and either $t_{i,i1}$ is a receiver of a global message or no node in $t_i$ is receiving a global message. This step schedules $t_{i,i1} \rightarrow t_{i,i2}$ in this phase.

Step 6: Use either the broadcast pattern or the rotate pattern for messages in $t_i \rightarrow t_j$, $i < j$ and $i \neq 0$.
/* scheduling of these global message would not affect the scheduling of local messages. */

**Figure 4. The global and local message assignment algorithm**

In Step 4, global messages in $t_i \rightarrow t_j$, $i > j$ and $j \neq 0$ are assigned using the broadcast scheme as follows: $t_{i,0} \rightarrow t_{j,0}, ..., t_{i,0} \rightarrow t_{j,|M_j|-1}, t_{i,1} \rightarrow t_{j,0}, ..., t_{i,1} \rightarrow t_{j,|M_j|-1}, t_{i,|M_i|-1} \rightarrow t_{j,0}, ..., t_{i,|M_i|-1} \rightarrow t_{j,|M_j|-1}$.

In Step 5, we schedule local messages in subtrees $t_i$, $t_i$, $1 \leq i < k$, in the phases for $t_i \rightarrow t_{i-1}$. Note that $|M_{i-1}| \geq |M_i|$ and there are $|M_i| \times |M_{i-1}|$ phases for messages in $t_i \rightarrow t_{i-1}$, which is more than the $|M_i| \times (|M_i| - 1)$ phases needed for local messages in $t_i$. There are some subtle issues in this step. First, all local messages are scheduled before assigning phases to global messages in $t_i \rightarrow t_j$, $1 \leq i < j$. The reason that global messages in $t_i \rightarrow t_j$, $1 \leq i < j$, do not affect the local message scheduling in subtree $t_n$, $1 \leq n < k$, is that all local messages in $t_n$ are scheduled in phases after the first phase for $t_0 \rightarrow t_n$ (since $|M_n| \times |M_{n-1}| \leq |M_0| \times |M_n|$) while phases for $t_i \rightarrow t_j$, $1 \leq i < j$, are all before that phase. Second, let us examine how exactly a communication $t_{i,i_2} \rightarrow t_{i,i_1}$ is scheduled. From Step 4, the receiver in $t_j \rightarrow t_i$, $j > i$, is organized such that, at phase $p$, $t_{i,(p-|M_0|\times(|M|-|M_0|)) \ mod \ |M_i|}$ is the receiver. From Step 1, receivers in $t_0 \rightarrow t_i$ are also aligned such that at phase $p$, $t_{i,(p-|M_0|\times(|M|-|M_0|)) \ mod \ |M_i|}$ is the receiver. Hence, in the phases for $t_i \rightarrow t_{i-1}$, either $t_{i,(p-|M_0|\times(|M|-|M_0|)) \ mod \ |M_i|}$ is a receiver of a global message or no node in $t_i$ is receiving a global message. Thus, at all phases in $t_i \rightarrow t_{i-1}$, we can assume that the designated receiver is $t_{i,(p-|M_0|\times(|M|-|M_0|)) \ mod \ |M_i|}$ at phase $p$. In other words, at phase $p$, $t_{i,(p-|M_0|\times(|M|-|M_0|)) \ mod \ |M_i|}$ can be scheduled as the sender of a local message. Now, consider the sender pattern in $t_i \rightarrow t_{i-1}$. Since $t_i \rightarrow t_{i-1}$ is scheduled using the broadcast pattern, each $t_{i,i_1}$ is sending in $|M_{i-1}|$ continuous phases. Since the receiving pattern covers every node, $t_{i,i_2} \in t_i$, in every $|M_i|$ continuous phases and $|M_{i-1}| \geq |M_i|$, there exists at least one phase where $t_{i,i_1}$ is sending a global message and $t_{i,i_2}$ is the designated receiver of a global message. Local message $t_{i,i_2} \rightarrow t_{i,i_1}$ is scheduled in this phase.

Finally, since all local messages are scheduled, we can use either the broadcast scheme or rotate scheme to realize messages in $t_i \rightarrow t_j$, $i < j$ and $i \neq 0$.

**Theorem:** The global and local message assignment algorithm in Figure 4 produces phases that satisfy the following conditions: (1) all messages in AAPC are realized in $|M_0| \times (|M| - |M_0|)$ phases; and (2) there is no contention within each phase.

*Proof*: From Lemma 2, all global messages are scheduled in $|M_0| \times (|M| - |M_0|)$ phases. Step 3 in the algorithm indicates that local messages in $t_0$ are scheduled in $|M_0| \times (|M_0| - 1)$ phases. In Step 5, all local messages in $t_i$ are scheduled in the phases allocated to communications in $t_i \rightarrow t_{i-1}$. Thus, all messages in AAPC are scheduled in $|M_0| \times (|M| - |M_0|)$ phases.

Lemma 4 shows that there is no contention among global messages in each phase. Since local messages in different subtrees cannot have contention and since in one phase, at most one local message in a subtree is scheduled, the contention can only happen between a global message and a local message inside a subtree. Yet, due to local message assignment done in steps 3 and 5 and from Lemma 3, all local messages have no contention with global messages. Thus, there is no contention within a phase. □

Table 4 shows the result of the global and local message assignment for the example in Figure 1. In this table, we can assume $t_{0,0} = n0$, $t_{0,1} = n1$, $t_{0,2} = n2$, $t_{1,0} = n3$, $t_{1,1} = n4$, and $t_{2,0} = n5$. We first determine the receiver pattern in $t_0 \rightarrow t_1$ and $t_0 \rightarrow t_2$. For messages in $t_0 \rightarrow t_1$, $t_{1,(p-9) \ mod \ 2}$ is the receiver at phase $p$, which means the receiver pattern from phase 0 to phase 5 are $t_{1,1}, t_{1,0}, t_{1,1}, t_{1,0}, t_{1,1}, t_{1,0}$. After that, the rotation pattern is used to realize all messages in $t_0 \rightarrow t_1$ and $t_0 \rightarrow t_2$. The results are shown in the second column in the figure. In the second step, messages in $t_1 \rightarrow t_0$ and $t_2 \rightarrow t_0$ are assigned. Messages in $t_2 \rightarrow t_0$ occupy the first round (first three phases). Since the sender pattern in the first round is $t_{0,0}$, $t_{0,1}$, and $t_{0,2}$, according to Table 3, the receiver pattern should be $t_{0,1}, t_{0,2}, t_{0,0}$. The receivers for $t_1 \rightarrow t_0$ are assigned in a similar fashion. After that, the broadcast pattern is used to realize both $t_1 \rightarrow t_0$ and $t_2 \rightarrow t_0$. In Step 3, local messages in $t_0$ are assigned in the first $3 \times 2 = 6$ phases according to the assignment of the sender and receiver of global messages in each phase. For example, in phase 0, local message $t_{0,1} \rightarrow t_{0,0}$ is scheduled since node $t_{0,0}$ is a sender of a global message and $t_{0,1}$ is a receiver of a global message. Note that the mapping in Table 3 ensures that all local messages in $t_0$ can be scheduled. In Step 4, $t_2 \rightarrow t_1$ is scheduled with a broadcast pattern. In Step 5, local messages in $t_1$ and $t_2$ are scheduled. The local messages in $t_1$ are scheduled in phases for $t_1 \rightarrow t_0$, that is, from phase 3 to phase 8. The alignment of the receivers in $t_0 \rightarrow t_1$ and $t_2 \rightarrow t_1$ ensures that each machine in $t_1$ appears as the designated receiver in every $|M_1| = 2$ phases starting from the first phase for $t_0 \rightarrow t_1$. Notice that in phase 6, the designated receiver is $t_{1,1}$. In $t_1 \rightarrow t_0$, each node in $t_1$ is the sender for $|M_0| = 3$ consecutive phases and the receiver pattern in $t_1$ covers every node in every 2 phases. All local messages in $t_1$ can be scheduled. In this particular example, message $t_{1,0} \rightarrow t_{1,1}$ is scheduled at phase 7 where $t_{1,0}$ is a (designated) receiver of a global message and $t_{1,1}$ is a sender of a global message, and $t_{1,1} \rightarrow t_{1,0}$ is scheduled at phase 4. Finally, in Step 6, we use the broadcast pattern for messages in $t_1 \rightarrow t_2$.

## 5. Implementation Issues

We develop an automatic routine generator that takes the topology information as input and automatically produces a topology-specific *MPI_Alltoall* routine that is built on top of MPI point-to-point primitives. The routine is intended to be used when the message size is large. The software currently works with LAM/MPI [8].

To be optimal, the AAPC phases must be separated to preserve the contention-free schedule. A simple way to achieve this is to add a barrier between each phase. Using barriers, however, would incur substantial synchronization overheads unless special hardware for the barrier operation such as the Purdue PAPERS [2] is available.

In our implementation, we use a pair-wise synchronization scheme. When two messages $a \rightarrow b$ in phase $p$ and $c \rightarrow d$ in phase $q$ have contention, $p < q$, the pair-wise synchronization makes sure that these two messages do not occur at the same time by introducing a synchronization message from node $a$ to node $c$ af-

**Table 4. Results of global and local message assignment for the cluster in Figure 1**

| | global messages | | | local messages | | |
|---|---|---|---|---|---|---|
| phase | $t_0 \rightarrow \{t_1, t_2\}$ | $t_1 \rightarrow \{t_2, t_0\}$ | $t_2 \rightarrow \{t_0, t_1\}$ | $t_0$ | $t_1$ | $t_2$ |
| 0 | $t_{0,0} \rightarrow t_{1,1}$ | $t_{1,0} \rightarrow t_{2,0}$ | $t_{2,0} \rightarrow t_{0,1}$ | $t_{0,1} \rightarrow t_{0,0}$ | | |
| 1 | $t_{0,1} \rightarrow t_{1,0}$ | $t_{1,1} \rightarrow t_{2,0}$ | $t_{2,0} \rightarrow t_{0,2}$ | $t_{0,2} \rightarrow t_{0,1}$ | | |
| 2 | $t_{0,2} \rightarrow t_{1,1}$ | | $t_{2,0} \rightarrow t_{0,0}$ | $t_{0,0} \rightarrow t_{0,2}$ | | |
| 3 | $t_{0,0} \rightarrow t_{1,0}$ | $t_{1,0} \rightarrow t_{0,2}$ | | $t_{0,2} \rightarrow t_{0,0}$ | | |
| 4 | $t_{0,1} \rightarrow t_{1,1}$ | $t_{1,0} \rightarrow t_{0,0}$ | | $t_{0,0} \rightarrow t_{0,1}$ | $t_{1,1} \rightarrow t_{1,0}$ | |
| 5 | $t_{0,2} \rightarrow t_{1,0}$ | $t_{1,0} \rightarrow t_{0,1}$ | | $t_{0,1} \rightarrow t_{0,2}$ | | |
| 6 | $t_{0,0} \rightarrow t_{2,0}$ | $t_{1,1} \rightarrow t_{0,0}$ | | | | |
| 7 | $t_{0,1} \rightarrow t_{2,0}$ | $t_{1,1} \rightarrow t_{0,1}$ | $t_{2,0} \rightarrow t_{1,0}$ | | $t_{1,0} \rightarrow t_{1,1}$ | |
| 8 | $t_{0,2} \rightarrow t_{2,0}$ | $t_{1,1} \rightarrow t_{0,2}$ | $t_{2,0} \rightarrow t_{1,1}$ | | | |

ter message $a \rightarrow b$. Message $c \rightarrow d$ is performed after node $c$ receives the synchronization message. Some synchronization messages may not be necessary as the ordering can be derived from other synchronization messages. Such synchronizations are referred to as *redundant synchronizations*. For example, assume that message $m1$ must synchronize with message $m2$ and with another message $m3$. If message $m2$ also needs to synchronize with message $m3$, then the synchronization from $m1$ to $m3$ can be removed.

Our implementation computes the required synchronizations as follows. For every communication at a phase, we check if a synchronization is needed for every other communication at later phases and build a dependence graph. After deciding all synchronization messages for all communications, we compute and remove redundant synchronizations in the dependence graph. In code generation, synchronization messages are added for all the remaining edges in the dependence graph. This way, the AAPC algorithm maintains a contention-free schedule while minimizing the number of synchronization messages.

## 6. Experiments

We evaluate the scheduling scheme by comparing our automatically generated routine with the original routine in LAM/MPI [8] and a recent improved *MPI_Alltoall* implementation in MPICH [18]. LAM/MPI implements all-to-all by simply posting all nonblocking receives and sends and then waiting for all communications to finish. Let $N$ denote the number of nodes in the system, and $i \rightarrow j$ denote the communication from node $i$ to node $j$. In this simple algorithm, the order of communications for node $i$ is $i \rightarrow 0$, $i \rightarrow 1$, ..., $i \rightarrow N-1$. The improved MPICH implementation uses different techniques and adapts based on the message size and the number of nodes in the system. For messages where $256 < msize \leq 32768$, a similar approach to that of LAM is used except that the order of communications for node $i$ is $i \rightarrow i+1$, $i \rightarrow i+2$, ..., $i \rightarrow (i+N-1)$ $mod$ $N$. For messages larger than 32768, and when the number of nodes is a power of two, MPICH uses a pairwise algorithm where node $i$ sends and receives from node $i \,\hat{}\, j$ at step $j$ $(1 \leq j < N)$. When the number of nodes is not a power of two, a ring algorithm is used. In this case, at step $j$, node $i$ sends to node $i+j$ and receives from node $i-j$. Both pairwise and ring algorithms finish AAPC in $N-1$ steps.

We use LAM/MPI 6.5.9 in the experiments. The MPICH implementation is slightly modified to work with LAM/MPI. The experiments are performed on a 32-node Ethernet switched cluster. The nodes of the cluster are Dell Dimension 2400 with a 2.8MHz P4 processor, 128MB of memory, and 40GHz of disk space. All machines run Linux (Fedora) with 2.6.5-1.358 kernel. The Ethernet card in each machine is Broadcom BCM 5705 with the driver from Broadcom. These machines are connected to Dell PowerEdge 2224 and Dell PowerEdge 2324 100Mbps Ethernet switches.

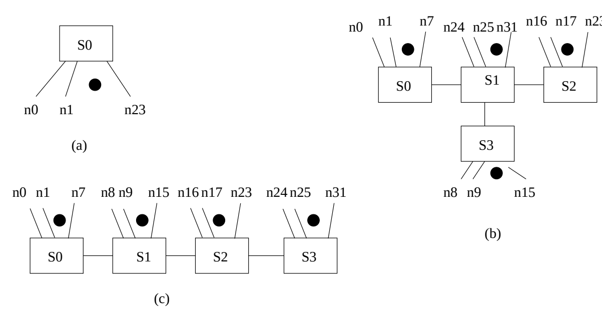



**Figure 5. Topologies used in the experiments**

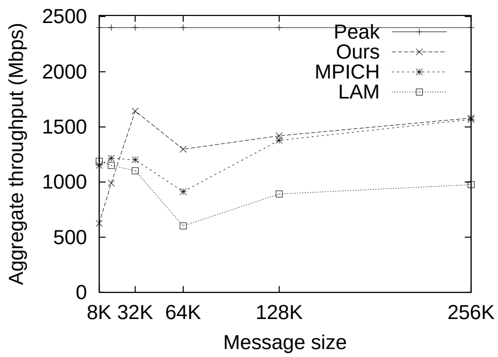
The topologies used in the experiments are shown in Figure 5. Part (a) of the figure is a 24-node cluster connected by a single 2324 switch. Both parts (b) and (c) represent a 32-node cluster where 8 nodes are connected to each of the 2224 switches. We will refer to these topologies as *topology (a)*, *topology (b)*, and *topology (c)*.

We compare our algorithm against LAM/MPI and MPICH in terms of the average completion or execution time and the actual aggregate throughput.

The completion times reported are the averages of three executions. In each execution, 10 iterations of *MPI_Alltoall* are measured and the average execution time for each invocation of the routine is recorded.

| msize | LAM | MPICH | Ours |
|---|---|---|---|
| 8KB | 29.7ms | 30.7ms | 56.5ms |
| 16KB | 61.4ms | 58.1ms | 71.4ms |
| 32KB | 128.2ms | 117.6ms | 86.0ms |
| 64KB | 468.8ms | 309.7ms | 217.7ms |
| 128KB | 633.7ms | 410.0ms | 398.0ms |
| 256KB | 1157ms | 721ms | 715ms |

(a) Completion time

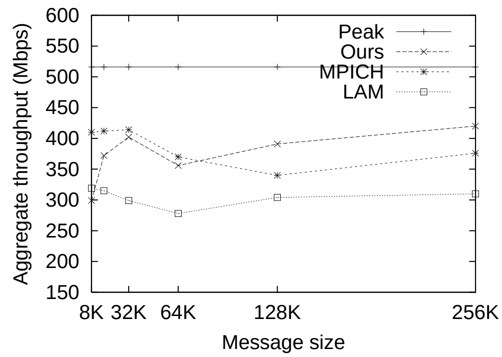



(b) Aggregate throughput

**Figure 6. Results for topology (a)**

| msize | LAM | MPICH | Ours |
|---|---|---|---|
| 8KB | 199ms | 155ms | 212ms |
| 16KB | 403ms | 308ms | 341ms |
| 32KB | 848ms | 613ms | 632ms |
| 64KB | 1827ms | 1374ms | 1428ms |
| 128KB | 3338ms | 2989ms | 2595ms |
| 256KB | 6550ms | 5405ms | 4836ms |

(a) Completion time




(b) Aggregate throughput

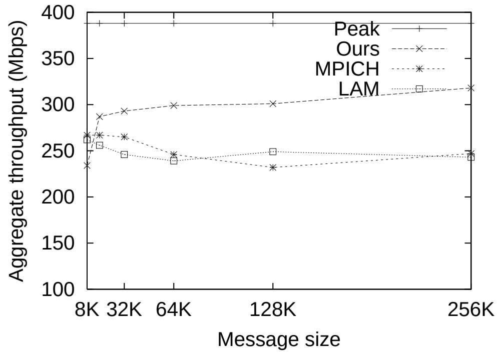
**Figure 7. Results for topology (b)**

Figures 6, 7, and 8 show the results for topologies (a), (b), and (c), respectively. Notice that for different topologies, the communication completion time is very different. This is because different topologies create different types of bottleneck links and different loads on the bottleneck links. The overall performance of LAM is poor, which can be attributed to the simple algorithm that does not perform any scheduling and results in severe network contention when the message size is large. MPICH has an advantage over LAM as it utilizes a number of different algorithms based on the message size. Although these algorithms perform a limited form of message scheduling, they do not consider the contention in the network links. Therefore, the performance of MPICH depends heavily on the network topology. As shown in the results in Figure 8 for topology (c), MPICH has a similar performance to LAM.

Unlike LAM and MPICH, our generated routine offers consistent better results when the message size is sufficiently large. This demonstrates the superiority of our algorithm in exploiting network bandwidths. To elaborate, consider the results for topology (a) in Figure 6. When message sizes are $32KB$ and $64KB$, the generated routine performs significantly better than LAM and MPICH. For example, when the message size is 64KB, the completion time for LAM is 468.8ms, 309.7ms for MPICH, and 217.7ms for our generated routine, which constitutes a speed up of 115% over LAM and 42.3% over MPICH. Note that when the message size is larger than $32KB$, the automatically generated routine uses a similar algorithm for this topology to the ring algorithm in MPICH. The difference is that pair-wise synchronizations are inserted between the phases in our algorithm to eliminate the contention in end nodes. Without the synchronizations, a limited form of node contention exists, which is shown in the experiment when the message sizes are $32KB$ and $64KB$. For message sizes $>$ 128KB, the contention in the switch becomes the dominant factor, and both the generated routine and MPICH perform similarly. Topologies (b) and (c) is different from topology (a) in that there are bottleneck links between switches, which indicates that the contention in the bottleneck link may have a larger impact than that in the switches. In this case, the generated routines, which use the scheduling scheme that eliminates link contention, have a larger advantage when the message size is large. For example, when the message size is 128KB, the generated routine achieves a speed up of 28.6% over LAM and 15.2% over MPICH on topology (b) and a speed up of 21% over LAM and 30% over MPICH on topology (c).

| msize | LAM | MPICH | Ours |
|---|---|---|---|
| 8KB | 242ms | 238ms | 271ms |
| 16KB | 495ms | 476ms | 443ms |
| 32KB | 1034ms | 958ms | 868ms |
| 64KB | 2127ms | 2061ms | 1700ms |
| 128KB | 4080ms | 4379ms | 3372ms |
| 256KB | 8375ms | 8210ms | 6396ms |

(a) Completion time




(b) Aggregate throughput

**Figure 8. Results for topology (c)**

# 7. Conclusion

In this paper, we introduce a message scheduling algorithm for AAPC on Ethernet switched clusters. We demonstrate that our AAPC algorithm can utilize network bandwidths more effectively than existing AAPC implementations in LAM/MPI and MPICH. The proposed AAPC algorithm can be applied to other networks with a tree topology.

# References


[1] S. Bokhari, "Multiphase Complete Exchange: a Theoretical Analysis," *IEEE Trans. on Computers*, 45(2), pages 220-229, February 1996.

[2] H. G. Dietz, T. M. Chung, T. I. Mattox, and T. Muhammad, "Purdue's Adapter for Parallel Execution and Rapid Synchronization: The TTL_PAPERS Design," *Technical Report*, Purdue University School of Electrical Engineering, January 1995.

[3] V. V. Dimakopoulos and N.J. Dimopoulos, "Communications in Binary Fat Trees," *International Conference on Parallel and Distributed Computing Systems*, Florida, USA, pages 383 - 388, Sept. 1995.

[4] A. Faraj and X. Yuan, "Message Scheduling for All–to–all Personalized Communication on Ethernet Switched Clusters," *Technial Report*, TR-041206, Department of Computer Science, Florida State University, Dec. 2004. Available at http://www.cs.fsu.edu/∼xyuan/alltoalltr.pdf

[5] S. Hinrichs, C. Kosak, D.R. O'Hallaron, T. Stricker, and R. Take, "An Architecture for Optimal All–to–All Personalized Communication," In *6th Annual ACM Symposium on Parallel Algorithms and Architectures*, pages 310–319, June 1994.

[6] S. L. Johnsson and C. T. Ho, "Optimum Broadcasting and Personalized Communication in Hypercubes," *IEEE Transactions on Computers*, Volumn 38, No. 9, pages 1249-1268, Sept. 1989.

[7] L. V. Kale, S. Kumar, and K. Varadarajan, "A Framework for Collective Personalized Communication," *International Parallel and Distributed Processing Symposium (IPDPS'03)*, p. 69a, April 2003.

[8] LAM/MPI Parallel Computing. http://www.lam-mpi.org/.

[9] C. C. Lam, C. H. Huang, and P. Sadayappan, "Optimal Algorithms for All–to–All Personalized Communication on Rings and two dimensional Tori," *Journal of Parallel and Distributed Computing*, 43(1):3-13, 1997.

[10] W. Liu, C. Wang, and K. Prasanna, "Portable and Scalable Algorithms for Irregular All–to–all Communication," *16th ICDCS*, pages 428-435, 1996.

[11] The MPI Forum. "The MPI-2: Extensions to the Message Passing Interface," July 1997. Available at http://www.mpi-forum.org/docs/mpi-20-html/mpi2-report.html.

[12] R. Ponnusamy, R. Thakur, A. Chourdary, and G. Fox, "Scheduling Regular and Irregular Communication Patterns on the CM-5," *Supercomputing*, pages 394-402, 1992.

[13] N.S. Sundar, D. N. Jayasimha, D. K. Panda, and P. Sadayappan, "Hybrid Algorithms for Complete Exchange in 2d Meshes," *International Conference on Supercomputing*, pages 181–188, 1996.

[14] D.S. Scott, "Efficient All–to–All Communication Patterns in Hypercube and Mesh topologies," *the Sixth Distributed Memory Computing Conference*, pages 398-403, 1991.

[15] A. Tam and C. Wang, "Efficient Scheduling of Complete Exchange on Clusters," *the ISCA 13th International Conference on Parallel and Distributed Computing Systems*, pages 111-116, August 2000.

[16] Andrew Tanenbaum, "Computer Networks," 4th Edition, 2004.

[17] R. Thakur and A. Choudhary, "All-to-all Communication on Meshes with Wormhole Routing," *8th International Parallel Processing Symposium (IPPS)*, pages 561-565, 1994.

[18] R. Thakur, R. Rabenseifner, and W. Gropp, "Optimizing of Collective Communication Operations in MPICH," *ANL/MCS-P1140-0304*, Mathematics and Computer Science Division, Argonne National Laboratory, March 2004.

[19] E. A. Varvarigos and D. P. Bertsekas, "Communication Algorithms for Isotropic Tasks in Hypercubes and Wraparound Meshes," *Parallel Computing*, Volumn 18, pages 1233-1257, 1992.