A General Approach for Tight Timing Predictions of Non-Rectangular Loops

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Rectangular versus Non-Rectangular Loop Nests

for (i = 0; i < 10; i++)
  for (j = 0; j < 10; j++)

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  for (j = 0; j < i; j++)
Goals

• Automatically calculate an accurate number of iterations for non-rectangular loop nests.

• Use a general approach that is not limited to nesting depth or dependences between index variables among loops in the nest.

• Integrate with an existing timing analyzer to obtain tight WCET and BCET of every loop.
Formulating the Number of Iterations as a Summation

• For a simple loop,

\[
\text{for } (i = a; \ i <= b; \ i++) \ldots
\]

we define the number of iterations as follows:

\[
I = \sum_{i=a}^{b} 1 = \begin{cases} 
  b - a + 1 & \text{if } a \leq b \\
  0 & \text{otherwise}
\end{cases}
\]

• Constraint on bounds is necessary due to zero-trip loops.
A Partially Zero-Trip Loop

\[
\text{for } (i=1; i<8; i++) \\
\quad \text{for } (j=i; j<3; j++) \\
I = \sum_{i=1}^{7} \begin{cases} 
3 - i & \text{if } i \leq 2 \\
0 & \text{otherwise}
\end{cases}
\]

- This summation equals 3, but a naive calculation would result in \(-7\).
Dealing with Nonunit Strides

for \( i = a; \ i \leq b; \ i += s \)...

- Summations involving nonunit strides are converted to uniform summations according to:

\[ I = \sum_{i=a}^{b,s} e = \sum_{i=0}^{\lfloor (b-a)/s \rfloor} e [i \leftarrow si + a] \]

- All free occurrences of \( i \) are replaced by \( si + a \).
- The resulting summation contains floor expressions, which can be rewritten as modulo operations.
Detecting the Absence of a Partially Zero Trip Loop

```
for (i = 0; i < 10; i++)
    for (j = i; j < 11; j++)
        for (k = i-3; k < j+8; k++)
```

we expand the initial value and limit:

\[
i - 3 = [0 - 3..9 - 3] = [-3..6]
\]
\[
j + 8 = [i + 8..10 + 8] = [8..18]
\]

• The ranges do not overlap, so the loop nest is not partially zero trip.
Conclusion

• Developed a general approach to count loop iterations as a nested summation.

• Timing analyzer formulates summation expression, evaluates the sum, and computes average number of iterations to compute tight WCET/BCET.

• Currently working on arbitrary nonunit strides within a loop nest.

• On-line demo available:
  http://www.cs.fsu.edu/~engelen/iternum.cgi