Recall that your solution must be typed via Latex. Please submit two copies of your paper to Canvas, one with name, and the other anonymous.

1. (25 points) Suppose that we throw $m$ balls at random into $n$ bins. For any $1 \leq i < j \leq m$, we say that the $i$-th and $j$-th balls create a collision if these two balls land into the same bin. Let $X$ be the total number of collisions. Compute $E[X]$.

2. (25 points) Consider the following recurrence:

\[
A_1 = 2 \\
A_n = \left(1 - \frac{2}{n+1}\right)A_{n-1} + 2, \text{ for any } n > 1.
\]

Using the summation factor, find a closed-form solution for $A_n$.

3. (25 points) A grocery store has coupons of $n$ colors. A customer buys $m$ coupons, each of a truly random color. Let $X$ be the number of coupon colors that the customer owns. Compute $E[X]$.

4. (25 points) Suppose that we want to sort an array $A[1:n]$ of $n$ distinct numbers. To improve the performance of Randomized Quicksort, instead of choosing a pivot at random from $\{A[1], \ldots, A[n]\}$, we sample three distinct elements $A[i], A[j], A[k]$ randomly from $\{A[1], \ldots, A[n]\}$, and then pick the pivot as the median of $A[i], A[j], A[k]$. Let $C_n$ be the expected running time for sorting an array of $n$ elements. Assume that sampling the pivot and partitioning $A[1:n]$ takes $n+1$ steps. Find a recurrence of $C_n$. (You don’t need to handle the base case $n \in \{0,1,2\}$. Just give the recurrence formula for $n > 2$.) You don’t have to solve this recurrence.