Recall that your solution must be typed via Latex. Please submit two copies of your paper to Canvas, one with name, and the other anonymous.

1. (35 points) Find a closed-form solution of the following recurrence using the repertoire method: $T_0 = 0$, and $T_n = T_{n-1} + n^3$ for every $n > 0$.

2. (35 points) Find a closed-form solution of the following sum using the perturbation method:

   $\sum_{k=1}^{n} k^2 \cdot 2^k$.

To simplify the solution, you’re allowed to use the following identity, which we already learned in class:

   $\sum_{k=1}^{n} k \cdot 2^k = (n - 1) \cdot 2^{n+1} + 2$.

3. (30 points) Suppose that we have $n$ circles (of possibly different radii) that all pass through the origin, but no two tangent at the origin. Also, except for the origin, no three circles pass through a common point. How many regions are created in the plane?