Recall that your solution must be typed via Latex. Also, remember that this is a group assignment; group size must be 2.

1. (70 points) You are given a sorted array of numbers where every value except one appears exactly twice; the remaining value appears only once. Design an efficient algorithm for finding which value appears only once.

Your solution should give a very high-level pseudocode, an informal explanation for the code, and a rigorous justification for the correctness of the algorithm. Your justification must work for the general case. If you simply consider some specific examples, it won’t count.

2. (70 points) Suppose that we want to merge two sorted arrays $A[1:n]$ and $B[1:n]$ into a fully sorted array. Show that any (comparison-based) algorithm for this task must use $\Omega(n)$ comparisons. Do not make extra assumption on what the algorithm does.

3. (60 points) Let $P$ be a convex polygon that is contained in another polygon $Q$, as illustrated in the left panel of Figure 1.1. We claim that the perimeter of $P$ is shorter than the perimeter of $Q$.

   a) (20 points) Prove the claim above for the special case that only one side of $P$ does not belong entirely to the perimeter of $Q$, as illustrated in the right panel of Figure 1.1.

   b) (40 points) Prove the claim above for the general case.

![Figure 1.1: Left: Illustration of polygons $P$ and $Q$, in general position. Right: Polygons $P$ and $Q$ in a special position, where only one side of $P$ doesn’t belong entirely to the perimeter of $Q.$]