Lecture 5: MAC

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The slides are loosely based on those of Prof. Mihir Bellare, UC San Diego.
Agenda

1. MAC and Authenticity

2. MAC Constructions

3. How to Construct Good MAC
The Need for Authenticity

Transfer $5 to account 12345

Transfer $1000 to account 99999

Classical encryptions (CTR, CBC) don’t provide authenticity
MAC Syntax

Key Gen

Canonical implementation:

\[ T = T_K(M) \]

MAC

Verify

Tag has fixed (short) length

0 or 1

Canonical implementation:

Return \( T = T_K(M) \)
MAC Usage

\[ T \leftarrow T_K(M) \]

\[ b \leftarrow V_K(M', T') \]
Formalizing Security

**MAC**

\[
\begin{align*}
\text{procedure Initialize()} & \\
& K \leftarrow \mathcal{K} \\
& \text{Return } \mathcal{T}_K(M) \\
\end{align*}
\]

\[
\begin{align*}
\text{procedure Tag}(M) & \\
& \text{Return } (T' = \mathcal{T}_K(M'))
\end{align*}
\]

\[
\text{Adv}_{\mathcal{T}}^\text{mac} (A) = \Pr[\text{MAC}_A^A \Rightarrow 1]
\]

Tag

Must never be queried
Replay Attack

Bob transfers $10 instead of $5!!

MAC wasn’t defined to handle replay attack.
Replay is best addressed as an add-on to standard msg authentication
Prevent Replay Attack Using Timestamp

\[ T \leftarrow \mathcal{T}_K (\text{Time}_A \| M) \]

Accept if:
\[ T = \mathcal{T}_K (\text{Time}_A \| M) \]
\[ |\text{Time}_A - \text{Time}_B| \leq \Delta \]

small interval
Prevent Replay Attack Using Counter

\[ T \leftarrow \mathcal{T}_K(\text{counter}_A \| M) \]
\[ \text{counter}_A \leftarrow \text{counter}_A + 1 \]

If \( T = \mathcal{T}_K(\text{counter}_B \| M) \)
\[ \text{counter}_B \leftarrow \text{counter}_B + 1 \]
accept

Counters need to be synchronized
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1. MAC and Authenticity
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3. How to Construct Good MAC
An Insecure Construction: Plain CBC-MAC

**Question:** Break CBC-MAC with a single Tag query
An Incorrect Fix of CBC-MAC

Exercise: Break this version using 3 Tag queries
An Good Construction: Encrypted CBC-MAC

Different key
Dealing with Fragmentary Data

**Solution:** Padding with $10^*$

**Question:** Can we instead use padding with $0^*$?

**Example:** Suppose that the block length is 16 bytes.

No padding $\rightarrow$ save bandwidth

**Answer:** No, can break this with a single Tag query
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1. MAC and Authenticity
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3. How to Construct Good MAC
PRF Is a Good MAC

**Intuition:** - A good MAC means the output should be unpredictable
- Random strings are unpredictable

**Question:** Given a good MAC $F$, construct $F'$ that is still a good MAC but has a trivial PRF attack.
PRF Extension

Blockcipher: Good PRF with small domain $\{0, 1\}^n$

$E_K$

How to extend the domain of a PRF?

$F_{K'}$

Want: Good PRF with large domain $\{0, 1\}^*$
Extending Domain: Carter-Wegman Paradigm

Condensing msg using a (keyed) hash

The hash needs to be **almost-universal**:

$$\Pr_{L \leftarrow \mathcal{L}}[h_L(M_1) = h_L(M_2)] \leq \epsilon \text{ for any } M_1 \neq M_2$$
Encrypted CBC via Carter-Wegman Paradigm

CBC-MAC is almost universal