GENERALIZE & SPECIALIZE

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Overview

- Generalize
- Specialize

Diagram with shapes and arrows indicating the relationships.
Agenda

1. Pythagorean Theorem

2. Tiling

3. Lines in the Plane
Generalizing Pythagorean Theorem

generalize
Similarity Means Proportional Area
Equivalence Between General and Special Cases

\[ b^2 \]

\[ a^2 \]

\[ \lambda b^2 \]

\[ \lambda c^2 \]

\[ \lambda a^2 \]
Equivalence Between General and Special Cases

Square

A Particular Shape

Every Shape
Specializing Pythagorean Theorem
Specializing Pythagorean Theorem
Specializing Pythagorean Theorem
Specializing Pythagorean Theorem
Specializing Pythagorean Theorem

Specialized Pythagorean Theorem:

\[ \text{ } \]
Agenda

1. Pythagorean Theorem

2. Tiling

3. Lines in the Plane
Tiling

Given a $n \times n$ floor with one missing cell, where $n$ is a power of 2

**Want**: Tile the floor using triminos
Given a \( n \times n \) floor with one missing cell, where \( n \) is a power of 2

**Want:** Tile the floor using triminos
Specialized Tiling

Let’s try to solve the problem when the missing tile is at a corner.
An Inductive Approach

The problem is easy for the base case

Assume that we can tile for $n \times n$
when the missing cell is at the corner

How can we tile for $2n \times 2n$?
How to Use Recursion?

Given $2n \times 2n$, how to get to $n \times n$?
How to Use Recursion?

Bring back $n \times n$ from $2n \times 2n$

We can tile the blue quadrant, and then?
Tiling

Recursion helps to tile each other quadrant
Get Back To General Case

The missing cell is not at the corner anymore.
An Inductive Approach

The problem is again easy for the base case.

Assume that we can tile for $n \times n$

for **any place** of the missing cell.

How can we tile for $2n \times 2n$?
Recursion can *magically* deal with the arbitrary position of the missing cell
Tiling

Recursion helps to tile each other quadrant
Agenda

1. Pythagorean Theorem

2. Tiling

3. Lines in the Plane
The Problem

There are $n$ lines on the plane, in “general positions”

How many regions do these lines divide the plane?

$L_n = ?$
Generalizing The Problem

There are $n$ planes in the space, in “general positions”

How many regions do these planes divide the space?
Specializing The Problem

There are $n$ points on a line, in “general positions”

How many segments do these points divide the lines?

$P_n = n + 1$
Guessing

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_n$</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$L_n$</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>11</td>
<td>16</td>
</tr>
</tbody>
</table>

$L_{n+1} = L_n + P_n = L_n + n + 1$
How To Prove It?

If we add a line into $n$ existing lines, where are the $n$ points?

$$L_{n+1} = L_n + P_n$$