Problem Set 5

Due April 9, Noon, 2024

1. (5 pts) Consider the following context-free grammar G:

    $S \rightarrow ABS \mid AB$

    $A \rightarrow Aa \mid a$

    $B \rightarrow bA$

    Determine which of the following strings are in L(G) and which are not. Give derivations for those that are in L(G) and give a very brief explanation for those that are not.

    (a) *aabaab*

    (b) *aaaaba*

    (c) *aabbaa*

    (d) *abaaba*

    (e) *abaababa*

2. (5 pts) Find a context-free grammars that will generate the following language:

    $L = \{a^n b^m c^k : k = n + m, n \geq 0, m \geq 0\}$

3. (10 pts) Consider the language $L = \{ww^R : w \in \{a,b\}^*\}$.

    Find a context-free grammar for the *complement* of L.

4. (10 pts) The grammar below is ambiguous:

    $E \rightarrow E + E \mid E * E \mid (E) \mid a \mid b$

    (a) Extend this grammar to include subtraction (-) and exponentiation ($\uparrow$).

    (b) Is this grammar ambiguous? If so give two different parse trees for some yield.

    (c) Construct an unambiguous grammar for your extended grammar. Make sure the precedence and associativity is as we would expect.

5. (5 pts) Eliminate λ productions from the following grammar. Use the appropriate algorithms and show your work.

    $S \rightarrow AaB \mid aaB$

    $A \rightarrow \lambda$

    $B \rightarrow bbA \mid \lambda$

6. (5 pts) Transform the following grammar into Chomsky normal form. Show your work.

    $S \rightarrow abABB$

    $A \rightarrow bB \mid \lambda$

    $B \rightarrow BAaa \mid Ab \mid \lambda$

7. (5 pts) Eliminate unit productions from the following grammar. Us the appropriate algorithms and show your work.

$$S \rightarrow XY$$
$$X \rightarrow A$$
$$A \rightarrow B \mid a$$
$$B \rightarrow b$$
$$Y \rightarrow T$$
$$T \rightarrow Y \mid c$$

8. (10 pts) Construct npda's that accept the following languages. You must use the approach as in my cot4420 lecture notes for defining npdas'.
   (a) $L = \{a^n bc^{n+m} : n, m \geq 0\}$
   (b) $L = \{a^n b^m : 2n \leq m \leq 4n\}$
   (c) $L = \{ww : w \in \{a, b\}^+\}$

9. (5 pts) These questions relate to properties of context-free languages:
   (a) Are context-free languages closed under intersection? Explain.
   (b) Are context-free languages closed under Kleene *? Explain.
   (c) Are deterministic context-free languages closed under union? Explain.