COT 5310 Theory of Automata and Formal Languages March 21, 2024

Problem Set 4

Due April 2, 2024

1. (10 pts) For this problem, you can use all the functions that have been shown to be primitive recursive in the text. Show that the following functions are primitive recursive. Hint: check functions that are known to be primitive recursive and can specifically help with determining, for example in (a), if x is even.

   (a) Let $H(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$

   (b) Let $J(x) = \begin{cases} x/2 & \text{if } x \text{ is even} \\ (x-1)/2 & \text{if } x \text{ is odd} \end{cases}$

2. (10 pts) p is called *a larger twin prime* if p and $p - 2$ are both primes. (5, 7, 13,19 are larger twin primes.) Let $T(0) = 0$, $T(n)$ the nth larger twin prime. It is widely believed that there are infinitely many larger twin primes. Assuming that this is true, prove that $T(n)$ is computable. You can use functions and predicates in your program that we have previously shown to be p.r. in the text.

   Hint, write a program $\mathscr{P}$ based on the assumption being true. Note that you can use any predicates in your program that have shown to be true in the text such as Prime (x). What would happen to your program if the assumption were to be false?

3. (15 pts) For this problem review what it means to be partially computable and review theorems in Chapter 4.3. Show the following functions are partially computable. Hint: use the $STP^{(1)}(x, y, t)$ function in your programs $\mathscr{P}$.

   (a) Let $H_1(x) = \begin{cases} 1 & \text{if } \Phi(x, x) \downarrow \\ \uparrow & \text{otherwise} \end{cases}$

   (b) Let $A = \{a_1, \ldots, a_n\}$ be a finite set such that $\Phi(a_i, a_i) \uparrow$ for $1 \leq i \leq n$, and let:

   $$H_2(x) = \begin{cases} 1 & \text{if } \Phi(x, x) \downarrow \\ 0 & \text{if } x \in A \\ \uparrow & \text{otherwise} \end{cases}$$

4. (10 pts) Determine the programs $\mathscr{P}_1$ such that #($\mathscr{P}_1$) = 575. Show your work.

5. (15 pts) Consider the following slightly modified program $\mathscr{P}$ version that we worked with in Notes-L4.1. Find #($\mathscr{P}$) for this program. You can use a calculator to get the final value. Show your work.

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[A]   X1 ← X1 – 1
      X2 ← X2 – 1
      If X1 ≠ 0 GOTO B
      Y ← Y + 1
      If Y ≠ 0 GOTO E
[B]   If X2 ≠ 0 GOTO A
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6. This problem will not be graded but will be covered in the solution set. However, it is important and useful to try to do the problem.
   Suppose someone claims they have a program $\mathscr{P}$ that computes HALT(x, x). Give a counterexample that proves that the claim is false. That is, give an input x_0 for which $\mathscr{P}$ gives the wrong answer.