COT 5310 Theory of Automata and Formal Languages February 12, 2024

Problem Set 3

Due February 20, 2024

1. (10 pts) For the program $\mathscr{P}$ below, write out a computation of $\mathscr{P}$ beginning with the snapshot (1, σ), where σ consists of the equations X = 2, Y = 0, Z= 0. What is $\Psi^{(1)}(2)$ for this program?

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[A]  If  X ≠ 0 GOTO B
     Z ← Z + 1
     If  Z ≠ 0 GOTO E
[B]  X ← X - 1
     Y ← Y + 1
     Z ← Z + 1
     If  Z ≠ 0 GOTO A
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2. (10 pts) Let P(x) be a computable predicate. Show that the function *f* defined by:

$$f(x_1, x_2) = \{ x_1 + x_2 \quad \text{if } P(x_1 + x_2) \\ \uparrow \quad \text{otherwise} \}$$

is partially computable. (Note, you need to write a program in $\mathscr{S}$ that implements this function).

3. (10) Let π be a computable permutation (i.e. one-one onto function) of *N* and let π^{-1} be the inverse of π, i.e., $\pi^{-1}(y) = x$ if and only if $\pi(x) = y$.
Show that π^{-1} is computable.

4. (5 pts) Let $\pi(x)$ be the number of primes $\leq x$. Show that $\pi(x)$ is primitive recursive. (You can use other functions that have been shown to be primitive recursive in the text.)

5. (10 pts) Let $R(x, t)$ be a primitive recursive predicate. Let:

$$g(x, y) = \max_{t \leq y} R(x, t)$$

that is, $g(x, y)$ is the largest value of t less than or equal to y for which $R(x, t)$ is TRUE; if there is none then $g(x, y) = 0$. Prove that $g(x, y)$ is primitive recursive.

6. (5 pts) Let us call a program *P* a *straight-line* program if it contains no (labeled or unlabeled) instructions of the form: IF V ≠ 0 GOTO L. Show by induction on the length of programs, that if the length of straight-line program *P* is *k*, then $\Psi^{1}(x) \leq k$ for all *x*.

7. (10 pts) Give a detailed argument that that the function x^{y} is primitive recursive. You can use the fact that $x + y$ and xy (x times y) were shown to be primitive recursive.

8. (10 pts) Let $\mathscr{C}$ be a PRC class and le g_1 and g_2 belong to $\mathscr{C}$. Show that if:

$$h_1(x, y, z) = g_1(z, y, x)$$
$$h_2(x) = g_2(x, x, x)$$

then h_1 and h_2 also belong to $\mathscr{C}$.