HALT (x,y) IS NOT COMPUTABLE

Predicate Halt(x,y) x is input, y is #P.

Definition Halt(x,y) is true if $\psi_P^{(1)}(x)$ is defined (has an output value)
that is, if P halts on input x.
Halt(x,y) is false $\psi_P^{(1)}$ is undefined.
that is P does not halt..

Thm 2.1 Halt(x,y) is not a computable predicate.

Suppose Halt(x,y) is a computable predicate.
That is, it will give True or False for the
program that implements Halt(x,y).

Consider following program P with #P = y_0.

[A] IF HALT(X,X) GOTO A.

This program implements the following

$$\psi_{y_0}(x) = \begin{cases} \text{undefined if Halt}(x,x) \\ 0 \text{ if } \sim \text{Halt}(x,x) \end{cases}$$

Note: From definition of Predicate Halt(x,y) [line 1]
Halt (x,y_0) is true if $\psi_P^{(1)}$ is defined.
$\Rightarrow$ if $\sim$ Halt(x,x). (by $\psi_{y_0}(x)$)
if Halt (x,y_0) is false the $\psi_P^{(1)}$ is undefined
$\Rightarrow$ ie Halt(x,x) from $\psi_{y_0}^{(1)}$ ..

Thus we have Halt$(x,y_0) \Leftrightarrow \sim$ Halt(x,x)
Replacing x by y_0, we get:
Halt$(y_0,y_0) \Leftrightarrow \sim$ Halt(y_0,y_0)