

[A] $X_1 \leftarrow X_1 - 1$ (I₁)
 $X_2 \leftarrow X_2 - 1$ (I₂)
 If $X_1 \neq 0$ GOTO B (I₃)
 $Y \leftarrow Y + 1$ (I₄)
 GOTO E (I₅)
 [B] If $X_2 \neq 0$ GOTO A (I₆)
 GOTO E (I₇)

#X₁ = 2, #X₂ = 4, #Y = 1, #A = 1, #B = 2, #E = 5

Note: $\langle x, y \rangle = 2^x (2y + 1) - 1$

#(I₁) = is $\langle a, \langle b, c \rangle \rangle = \langle 1, \langle 2, 1 \rangle \rangle = \langle 1, 11 \rangle = 45$

#(I₃) = $\langle 0, \langle 4, 1 \rangle \rangle = \langle 0, 47 \rangle = 94$

Given # (I₃) = 94 find a, b, c, values

$x = l(z) =$ largest number such that $2^x \mid z + 1$

$y = r(z) =$ solution of $2y + 1 = (z + 1) / 2^x$

$a = l(94) = 0, \langle b, c \rangle = r(94) = 47$

$b = l(47) = 4, c = r(47) = 1$

$a = 0$ implies instruction is unlabeled.

$c = 1$ implies variable is $c + 1 = r(r(94)) + 1 = 2 = X_1$

$b = 4$ implies label is $b - 2 = l(r(94)) - 2 = 4 - 2 = B$

#(I₂) = $\langle 0, \langle 2, 3 \rangle \rangle = \langle 0, 27 \rangle = 54$

#(I₄) = $\langle 0, \langle 1, 0 \rangle \rangle = \langle 0, 1 \rangle = 2$

We will consider the program \mathcal{P} consisting of the 4 instructions computed above.

$\#\mathcal{P} = [\#(I_1), \#(I_2), \#(I_3), \#(I_4)] - 1 = [45, 54, 94, 2] - 1 = 2^{45} \times 3^{54} \times 5^{94} \times 7^2 - 1$

We consider \mathcal{U}_2 that computes $\Phi^{(2)}(X_1, X_2, X_3)$ where X_3 is $\#\mathcal{P}$ above.

$Z \leftarrow X_3 + 1$ Hence $Z = \#\mathcal{P} + 1 = [\#(I_1), \#(I_2), \#(I_3), \#(I_4)] = 2^{45} \times 3^{54} \times 5^{94} \times 7^2$

$S \leftarrow \prod_{(i=1 \text{ to } 2)} (p_{2i})^{X_i}$ Hence $S = 3^{X_1} \times 7^{X_2} = [0, X_1, 0, X_2]$ which is the initialization of input values. Note that the value of all other variables (including Y) is initially 0 and the value of Y is obtained by the exponent of 2 in the product S.

$K \leftarrow 1$