Pairing #s. & Function

Compute $z = (2^x)(2y+1) - 1$

	$x=0$	$x=1$	$x=2$	$x=3$	$x=4$
$y=0$	0	1	3	7	15
$y=1$	2	5	11	23	47
$y=2$	4	9	19	39	79
$y=3$	6	13	27	55	
$y=4$	8	17	35	71	
$y=5$	10	21	43	87	
$y=6$	12	25	51	103	
$y=7$	14	29	59		
$y=8$	16	33	67		
$y=9$	18	37	75		
$y=10$	20	41	83		
$y=11$	22	45	91		
$y=12$	24	49	99		
$y=13$	26	53	107		
$y=14$	28	57	115		
$y=15$	30	61	123		

Pairing Functions

Let $\langle x, y \rangle = z$

Then there is a <u>unique</u> solution x, y for the equation $\langle x, y \rangle = z$

Note that $\langle x, y \rangle + 1 = z + 1 = (2^x)(2y+1)$

Let x be the largest number such that

$2^x \mid z+1$

Note that there is a unique solution x for this.

Then $2y + 1 = \underbrace{(z+1)/2^x}_{\text{this must be an odd \#}}$

hence a unique value of y results from the equation.

Example Let $z = 20$ $z + 1 = \cancel{20}\ 21$. $2^x \mid 21$?

$x = 0$, $\therefore 2y + 1 = 21/1$

$\therefore y = 10$

$\therefore \langle 0, 10 \rangle = z = 20$

Example Let $z = 59$ $z+1 = 60$ $2^x \mid 60$ $x = 2$

$\therefore 2y+1 = 60/4 = 15$

$y = 7$

$\therefore (2, 7) = z = 59$

The n^{th} Prime # is P.R.

$$P_{n+1} = \min_{t \le P_n!+1} [\text{ Prime } t \text{ \& } t > P_n]$$

Example $(P_n)! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \cdots P_n$. Say $P_7 = 31$

$P_n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdots\cdots 31$ $\left[\frac{7}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} \right.$

$(P_n)!+1 =$ --Huge # $+ 1$

Note that $P_n!+1$ is not divisible by any prime $P_1 \cdots P_n$. why?

$$\frac{(P_n)!+1}{P_i} = k + \frac{1}{P_i} \quad i \le n$$

$\therefore$

(1) $(P_n)!+1$ is prime.

or $\exists$ some prime # $> P_n$ that divides $(P_n)!+1$.

$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot \underline{7} = 5040$

$7!+1 = 5041. = 71^2$

$$\boxed{\text{Next prime is } \min_{t \le 5041}}$$

Note Interesting

$7!+1$ is a perfect square 71^2

$4!+1 = 25 = 5^2$

$5!+1 = 121 = 11^2$

Any others?

Gödel Numbers

Given a sequence $(a_1, \dots a_n)$
The Gödel number written $[a_1, a_2, \dots a_n]$

$$[a_1, a_2, \dots a_n] = \prod_{i=1}^{n} p_i^{a_i}$$

where p_i is the i^{th} prime #
$p_0 = 0$, $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $p_4 = 7$ $p_5 = 11$, ...

$$[a_1, \dots a_n] = [b_1, \dots b_n]$$

then $a_i = b_i$
fundamental theorem of arithmetic
Note tho

$$[a_1, \dots a_n, 0] = [a_1 \dots a_n]$$

Empty sequence $[\ ] = 1$.
↳ length 0.

$$x = [a_1, \dots a_n]$$

$(x)_i = a_i$

$$(x)_i = \min_{t \le x} (\sim p_i^{t+1} \mid x)$$

Example $[3, 2, 0, 1] = 2^3 \cdot 3^2 \cdot 5^0 \cdot 7^1 = 504 = x$

$x_1 = \min_{t \le 504} (\sim 2^{t+1} \mid 504) \Rightarrow t = 3$

$x_3 = {}_{t \le 504} (\sim 5^{t+1} \mid 504) \Rightarrow t = 0$

$x_5 = {}_{t \le 504} (\sim 11^{t+1} \mid 504) \Rightarrow t = 0$

$x_4 = 1$ $(\sim 7^{t+1} \mid 504)$

$Lt(x)$

$$Lt(x) = \min_{i \le x} \left[(x)_i \neq 0 \ \& \ (\forall j_{\le x} : j \le i \vee (x)_j = 0) \right]$$

$$Lt(504) = \min_{i \le 504} [\qquad] = 4$$

Note that $(x)_1 \neq 0$ but $\underset{\sim}{x_4} \neq 0$ & $\underset{\sim}{4} \nleq 1$

This makes sure length is the longest ~~shortest~~ sequence ending in a nonzero value.