Advanced Algorithms	Due: September 3, 2009

Problem Set 1

Name: Your Name — *Collaborated with: Partner's Name*

- You have to answer only 2 out of 24 questions.
- The particular exercises you have to solve will be determined using $f_1()$ and $f_2()$ as specified below.

Example: My fsu email address: pkumar@fsu.edu
First 2 characters in hex: **pk** gives 706B.
$f_1() = (706B \mod 3D) \mod 18 + 1 = (30 \mod 18) + 1 = 0 + 1 = 1$. In Decimal = 1.
$f_2() = (f_1() + C) \mod 18 = D$. Converted to Decimal = 13.

Exercise 1

Suppose the people who own page 3 in the web of Figure 1 are infuriated by the fact that its importance score computed using formula(2.1), is lower than the score of page 1. In an attempt to boost page 3's score,they create a page 5 that links to page 3; page 3 also links to page 5. Does this boost page 3's score above that of page 1?

Solution:

insert solution here

Exercise 13

Show more generally that $(\mathbf{A}^p)_{ij} > 0$ iff page i can be reached from page j in exactly p steps.

Solution:

insert solution here. Some sample lines that might help when you typeset the math, don't forget to remove them after you are done...
$x_k \geq 0$ and $x_j > x_k$ means page j is more important than page k.

$x_1 = x_3 + x_4$

$x_1 = \frac{x_3}{1} + \frac{x_4}{2}$

Let $L_k \subset \{1, 2, \ldots, n\}$ denote the set of pages with a link to page k. Then:

$$x_1 = x_3/1 + x_4/2 \tag{1}$$
$$x_2 = x_1/3 \tag{2}$$
$$x_3 = x_1/3 + x_2/2 + x_4/2 \tag{3}$$
$$x_4 = x_1/3 + x_2/2 \tag{4}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

We seek an eigenvector $\mathbf{x}$ for eigenvalue 1. Let $V_1(\mathbf{A})$ be the eigenspace for the eigenvalue 1 of the column stochastic matrix $\mathbf{A}$. dimension$(V_1(\mathbf{A})) = 2$ in this case.

$$\mathbf{x} = [1/2, 1/2, 0, 0, 0]^T$$
$$\mathbf{y} = [0, 0, 1/2, 1/2, 0]^T$$

Web W consisting of r connected components forces dimension$(V_1(\mathbf{A}))\geq r$.

$$\begin{bmatrix} \mathbf{A}_1 & 0 \\ 0 & \mathbf{A}_2 \end{bmatrix} \begin{pmatrix} \mathbf{v}_1 \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbf{v}_1 \\ 0 \end{pmatrix}$$
$$\begin{bmatrix} \mathbf{A}_1 & 0 \\ 0 & \mathbf{A}_2 \end{bmatrix} \begin{pmatrix} 0 \\ \mathbf{v}_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{v}_2 \end{pmatrix}$$

Let $\mathbf{S}$ denote an $n \times n$ matrix with all entries $1/n$.

Lemma 1: *If $\mathbf{M}$ is positive and column stochastic, then any eigenvector in $V_1(\mathbf{M})$ has all positive or all negative components.*

Lemma 2: *Let $\mathbf{v}$ and $\mathbf{w}$ be linearly independent vectors in $\mathbf{R}^m, m \geq 2$. Then for some values of s and t, the vector $\mathbf{x} = s\mathbf{v} + t\mathbf{w}$ has both positive and negative components.*

A straightforward induction shows: $||\mathbf{M}^k\mathbf{v}||_1 \leq c^k||\mathbf{v}||_1$.