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CIS 5930, Spring 2013

Assignment #3
To be discussed in class on Monday, Feb 15th.

Each of the problems should be solved on a separate sheet of paper to facilitate grading. Please don't wait until the last minute to look at the problems.

Problem 1
Given a simple polygon P with n vertices and a point p inside it, show how to compute the region inside P that is visible from P. Make your solution as efficient as you can.

Problem 2
Given n lines in the plane, compute the upper chain of those lines in $O(nlogn)$ time. Assume that no two intersections have the same x-coordinate and all lines have different slopes. You are not allowed to use duality.

Problem 3 A slab is a region between two parallel lines. The width of a point set P, denoted by $w(P)$ is the width of the smallest slab that contains all the points in P. Show that $w(P) = w(\text{ConvexHull}(P))$. Compute the width of P in linear time given the convex hull of P in counter-clockwise order.

Problem 4
The objective of this problem is to consider an alternative approach for an $O(n \log h)$ time algorithm for computing the upper hull of a set of points in the plane. Given two nonempty planar point sets A and B, which lie on opposite sides of a vertical line, the upper tangent of A and B is defined to be a nonvertical line T that passes through one point $a \in A$ and one point $b \in B$ such that no point of $A \cup B$ lies above this line. (See Figure 1.) It is a fact (which you need not prove) that the upper tangent exists and is unique.

Consider the following variation of the divide and conquer convex hull algorithm, which given an n-element planar point set P computes its upper hull $H(P)$.

1. If $|P| \leq 3$, then compute the upper convex hull by brute force in $O(1)$ time and return.
2. Otherwise, partition the point set P into two sets A and B, where A consists of $\lfloor n/2 \rfloor$ the points with the lowest x-coordinates and B consists of $\lceil n/2 \rceil$ of the points with the highest x-coordinates.
3. Compute the upper tangent T between A and B and let $a \in A$ and $b \in B$ be the contact point with these sets. (Note: This is done without knowing the upper hulls.)

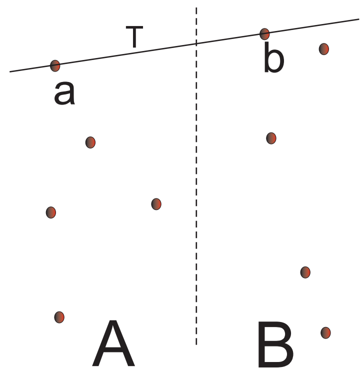

Figure 1: The upper hull of point sets A and B

4. Remove from A and B all the points whose x-coordinates lies strictly between a_x and b_x. (Do not remove a and b.) Let A' and B' be the respective subsets of remaining points.

5. Recursively compute the upper hulls $H(A')$ =`upperHull`(A') and $H(B')$ =`upperHull`(B').

6. Merge the two upper convex hulls $H(A')$ and $H(B')$ along with the segment ab to form the final upper hull.

The main difference between this algorithm and the divide-and-conquer algorithm given in class is that this algorithm computes the upper tangent before it computes the two hulls, rather than after. You may assume (without proving) that the computations of Steps (2) and (3) can be performed in $O(n)$ time, where $n = |P|$. (Step (2) follows from the well-known fast median algorithm and we will discuss Step (3) later this semester.) Using these facts, show that the algorithm described here takes $O(n \log h)$ time, where h is the number of points in the final upper convex hull.
Extra Credit : Implement this algorithm for your convex hull implementation and engineer it to be faster than Graham Scan or QuickHull at least for uniformly distributed points.