Minimum Cover of F

Minimal Cover for a Set of FDs

- ***Minimal cover*** **G for a set of FDs F:**
  - Closure of F = closure of G.
  - Right hand side of each FD in G is a single attribute.
  - If we modify G by deleting an FD or by deleting attributes from an FD in G, the closure changes.
- **Intuitively, every FD in G is needed, and ``*as small as possible*'' in order to get the same closure as F.**
- **e.g., A → B, ABCD → E, EF → GH, ACDF → EG has the following minimal cover:**
  - A → B, ACD → E, EF → G and EF → H
- **M.C. implies Lossless-Join, Dep. Pres. Decomp!!!**
  - Start with M.C. of F, do the decomposition from last slide

Functional dependencies

Our goal:

given a set of FD set, F, find an alternative FD set, G that is:

- smaller
- equivalent

Bad news:

Testing F=G (F+ = G+) is computationally expensive

Good news:

Canonical Cover algorithm:

given a set of FD, F, finds minimal FD set equivalent to F

Minimal: can’t find another equivalent FD set w/ fewer FD’s

Canonical Cover Algorithm

Given:

$F = \{ A \rightarrow BC,$
$B \rightarrow CE,$
$A \rightarrow E,$
$AC \rightarrow H,$
$D \rightarrow B\}$

- $F_c = F$
- No G that is equivalent to F and is smaller than Fc

Determines canonical cover of F:

$F_c = \{ A \rightarrow BH,$
$B \rightarrow CE,$
$D \rightarrow B\}$

Another example:

$F = \{ A \rightarrow BC,$
$B \rightarrow C,$
$A \rightarrow B,$
$AB \rightarrow C,$
$AC \rightarrow D\}$

⟶ CC Algorithm ⟶

$F_c = \{ A \rightarrow BD,$
$B \rightarrow C\}$

Canonical Cover Algorithm

Basic Algorithm

ALGORITHM CanonicalCover (X: FD set)
BEGIN
REPEAT UNTIL STABLE
(1) Where possible, apply UNION rule (A's axioms)
(e.g., A →BC, A→CD becomes A→BCD)
(2) remove "extraneous attributes" from each FD
(e.g., AB→C, A→B becomes
A→B, B→C
i.e., A is extraneous in AB→C)

Extraneous Attributes

(1) Extraneous is RHS?

e.g.: can we replace $A \rightarrow BC$ with $A \rightarrow C$?

(i.e. Is B extraneous in $A \rightarrow BC$?)

(2) Extraneous in LHS ?

e.g.: can we replace $AB \rightarrow C$ with $A \rightarrow C$?

(i.e. Is B extraneous in $AB \rightarrow C$?)

Simple but expensive test:

1. Replace $A \rightarrow BC$ (or $AB \rightarrow C$) with $A \rightarrow C$ in F

$$F2 = F - \{A \rightarrow BC\} \cup \{A \rightarrow C\}$$

or

$$F - \{AB \rightarrow C\} \cup \{A \rightarrow C\}$$

2. Test if $F2+ = F+$?

if yes, then B extraneous

Extraneous Attributes

A. RHS: Is B extraneous in A →BC?

step 1: F2 = F - {A →BC} U {A →C}
step 2: F+ = F2+ ?

To simplify step 2, observe that F2+ ⊆ F+

Why? Have effectively removed A→B from F i.e., not new FD's in F2+)

When is F+ = F2+ ?

Ans. When (A→B) in F2+

Idea: if F2+ includes: A→B and A→C, then it includes A→BC

Extraneous Attributes

B. LHS: Is B extraneous in A B→C ?

step 1: F2 = F - {AB →C} U {A →C}
step 2: F+ = F2+ ?

To simplify step 2, observe that F+ ⊆ F2+

Why? A→C "implies" AB→C. therefore all FD's in F+ also in F2+. i.e., there may be new FD's in F2+)

But AB→C does not "imply" A→C

When is F+ = F2+ ?

Ans. When (A→C) in F+

Idea: if F+ includes: A→C then it will include all the FD's of F2+.

Extraneous attributes

A. RHS :

Given F = {A→BC, B→C} is C extraneous in A →BC?

why or why not?

Ans: yes, because

A→C in { A→B, B→C}+

Proof. 1. A→ B

2. B →C

3. A→C transitivity using Armstrong's axioms

Extraneous attributes

B. LHS :

Given F = {A→B, AB→C} is B extraneous in AB →C?

why or why not?

Ans: yes, because

A→C in F+

Proof. 1. A→ B

2. AB →C

3. A→C using pseudotransitivity on 1 and 2

Actually, we have AA→C but {A, A} = {A}

Canonical Cover Algorithm

ALGORITHM CanonicalCover (F: set of FD's)
BEGIN
REPEAT UNTIL STABLE
(1) Where possible, apply UNION rule (A's axioms)

(2) Remove all extraneous attributes:
a. Test if B extraneous in A→ BC
(B extraneous if
(A→B) in (F - {A→BC} U {A→C})+)
b. Test if B extraneous in AB→C
(B extraneous in AB→C if
(A→C) in F+)

Canonical Cover Algorithm

Example: determine the canonical cover of

$F = \{A \rightarrow BC, B \rightarrow CE, A \rightarrow E\}$

Iteration 1:

a. $F = \{ A \rightarrow BCE, B \rightarrow CE\}$

b. Must check for upto 5 extraneous attributes

- B extraneous in $A \rightarrow BCE$? No
- C extraneous in $A \rightarrow BCE$?

  yes: $(A \rightarrow C)$ in $\{ A \rightarrow BE, B \rightarrow CE\}$

  1. $A \rightarrow BE$ -> 2. $A \rightarrow B$ -> 3. $A \rightarrow CE$ -> 4. $A \rightarrow C$
- E extraneous in $A \rightarrow BE$?

Canonical Cover Algorithm

Example: determine the canonical cover of

F = {A →BC, B→CE, A→E}

Iteration 1:

a. F = { A→BCE, B→CE}

b. Must check for upto 5 extraneous attributes

- B extraneous in A→BCE? No
- C extraneous in A → BCE? Yes
- E extraneous in A→BE?

    1. A→B -> 2. A→CE -> A→E

- E extraneous in B→CE No
- C extraneous in B→CE No

Iteration 2:

a. F = { A → B, B→ CE}

b. Extraneous attributes:

    - C extraneous in B → CE No
    - E extraneous in B →CE No

DONE

Canonical Cover Algorithm

Find the canonical cover of

$$F = \{ A \rightarrow BC, \; B \rightarrow CE, \; A \rightarrow E, \; AC \rightarrow H, \; D \rightarrow B \}$$

Ans: $F_c = \{ A \rightarrow BH, \; B \rightarrow CE, \; D \rightarrow B \}$

Canonical Cover Algorithm

Find two different canonical covers of:

$F = \{ A \rightarrow BC, \quad B \rightarrow CA, \quad C \rightarrow AB \}$

Ans:

$Fc1 = \{ A \rightarrow B, B \rightarrow C, C \rightarrow A \}$

and

$Fc2 = \{ A \rightarrow C, B \rightarrow A, C \rightarrow B \}$