

MAD 3105 PRACTICE TEST 1

Information about the midterm:

- There are 7 problems, but many of the problems have multiple parts. 4 questions require proof. The rest of the questions are short answer or true/false.
- When a proof specifies a method, you must use that method to receive full credit.
- On proofs that do not specify a method you may use any method.
- On single, short answer problems that do not specifically say to explain no explanation is required.
- On problems that say to explain or show work, you must do so to receive full credit. However, you do not have to give a formal proof. Show your work in the spaces provided on the exam or *clearly* indicate where work is shown.
- The problems on the exam are about the same level of difficulty as the problems on this review.
- The topics covered on the exam are a subset of the topics covered on this review.
- There is no time limit on the exam other than your schedule and the testing center's hours. The exam is designed to take between approximately 1-1.5 hours.
- No calculators or material other than writing utensils and scrap paper will be permitted. The testing center will either provide scrap paper or check scrap paper provided by the student. Please check with your testing center on their policy. The exam will have spaces in which you should show your work. Work done on your scrap paper will not be graded unless you *clearly indicate* within the space provided on the exam that there is work that should be considered on the scrap paper and *clearly label* the work on the scrap paper.

1. Let R be the relation defined below. Determine which properties, reflexive, ir-reflexive, symmetric, antisymmetric, transitive, the relation satisfies. Prove each answer.
 - (a) R is the relation on a set of all people enrolled in courses at FSU given by two people a and b are such that $(a, b) \in R$ if and only if a and b are enrolled in the same course at FSU.
 - (b) R is the relation on $\{a, b, c\}$, $R = \{(a, b), (b, a), (b, b), (c, c)\}$

- (c) R is the relation on the set of positive integers given by mRn if and only if $\gcd(m, n) > 1$.
- (d) R is the relation on the set of positive real numbers given by xRy if and only if x/y is a rational number.
- (e) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function $f(x) = \lfloor x \rfloor$. Define the relation R on the set of real numbers by $R = \text{graph}(f)$.
- (f) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function $f(x) = \lfloor x \rfloor$. Define the relation R on the set of real numbers by aRb iff $f(a) = f(b)$.
2. Let R be the relation $R = \{(a, c), (b, b), (b, c), (c, a)\}$ and S the relation $S = \{(a, a), (a, b), (b, c), (c, a)\}$ is a relation on $A = \{a, b, c\}$.
- (a) Find R^2 .
- (b) Find $S \circ R$.
- (c) Find R^{-1} .
3. The matrix below is the matrix for a relation.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

- (a) Which of the following properties does the relation satisfy: reflexive, irreflexive, symmetric, antisymmetric, asymmetric, transitive?
- (b) Find the matrix that represents R^{-1} .
- (c) Find the matrix that represents R^2 .
- (d) Find the matrix that represents $r(R)$.
- (e) Find the matrix that represents $s(R)$.
- (f) Find the matrix that represents $t(R)$.
4. Let A be a set and let R and S be relations on A . If R and S satisfy the property given, does the relation given have to satisfy the same property? Prove or disprove each answer.
- (a) Reflexive, $R \cup S$
- (b) Reflexive, R^{-1}
- (c) Symmetric, $t(R)$
- (d) Symmetric, $R \circ S$
- (e) Antisymmetric, $R \oplus S$
- (f) Antisymmetric, R^n for any positive integer n
- (g) Transitive, $r(R)$
- (h) Transitive, R^{-1}
- (i) Equivalence Relation, $R - S$
- (j) Equivalence Relation, R^n for any positive integer n
- (k) Partial Order, $R \circ S$
- (l) Partial Order, $R \oplus S$
5. Let R_1 be a relation from S to T and let R_2 be a relation from T to U . Prove $(R_2 \circ R_1)^{-1} = R_1^{-1} \circ R_2^{-1}$.

6. Let $A = \{1, 2, 3, 4, 5\}$ and let R_1 be the relation give by $(n, m) \in R_1$ iff $n \equiv m \pmod{3}$.
- Give the Boolean matrix for the relation.
 - Draw the digraph for the relation.
7. True or false, prove or disprove: If R and S are relations on A , then
- $r(R \cap S) = r(R) \cap r(S)$.
 - $r(R - S) = r(R) - r(S)$.
 - $s(R \cup S) = s(R) \cup s(S)$.
 - $s(R \circ S) = s(R) \circ s(S)$.
 - $t(R \cup S) = t(R) \cup t(S)$.
 - $t(R \circ S) = t(R) \circ t(S)$.
8. Let R be the relation on the set of all integers given by nRm if and only if $nm < 0$.
- Describe the reflexive closure of R . Begin with the expression “ $(m, n) \in r(R)$ if and only if”
 - Describe the symmetric closure of R
 - Describe the transitive closure of R
9. Which of the following relations are equivalence relations. For the relations that are equivalence relations find the equivalence classes. For the ones that are not equivalence relations name the property(ies) that fails.
- The relation R on the set of Computer Science majors at FSU where aRb iff a and b are currently enrolled in the same course.
 - The relation R on the set of integers where $(m, n) \in R$ if and only if $mn \equiv 2 \pmod{2}$.
 - The relation R on the set of ordered pairs of integers where $(a, b)R(c, d)$ iff $a = c$ or $b = d$
 - Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x) = \lceil x \rceil$. Define the relation R on \mathbb{R} by $(x, y) \in R$ if and only if $f(x) = f(y)$.
 - The relation R on the set of all subsets of $\{1, 2, 3, 4\}$ where SRT means $S \subseteq T$
 - The relation R on the set of positive integers where $(m, n) \in R$ if and only if $\gcd(m, n) = \max\{m, n\}$.
 - Let $G = (V, E)$ be a simple graph. Let R be the relation on V consisting of pairs of vertices (u, v) such that there is a path from u to v or such that $u = v$.
 - The relation R on the set of ordered pairs $\mathbb{Z}^+ \times \mathbb{Z}^+$ of positive integers defined by

$$(a, b)R(c, d) \Leftrightarrow a + d = b + c.$$

10. Let R be the relation on the set of ordered pairs of positive integers such that $(a, b)R(c, d)$ if and only if $ad = bc$.
- Prove R is an equivalence relation.
 - Find the equivalence class of (a, b) where $(a, b) \in \mathbb{Z}^+ \times \mathbb{Z}^+$:
11. Let R be the relation on the set \mathbb{R} real numbers defined by xRy iff $x - y$ is an integer. Prove that R is an equivalence relation on \mathbb{R} .
12. Suppose $A = \{2, 4, 5, 6, 7, 10, 18, 20, 24, 25\}$ and R is the partial order relation $(x, y) \in R$ iff $x|y$.

- (a) Draw the Hasse diagram for the relation.
 (b) Find all minimal elements.
 (c) Find all maximal elements.
 (d) Find all upper bounds for $\{6\}$.
 (e) Find all lower bounds for $\{6\}$.
 (f) Find the least upper bound for $\{6\}$.
 (g) Find the greatest lower bound for $\{6\}$.
 (h) Find the least element.
 (i) Find the greatest element.
 (j) Is this a lattice?
13. Suppose $A = \{2, 3, 4, 5\}$ has the usual “less than or equal” order on integers. Find each of the following for the case where R is the lexicographic order relation on $A \times A$ and where R is the product order relation on $A \times A$.
- (a) Draw the Hasse diagram for the relation.
 (b) Find all minimal elements.
 (c) Find all maximal elements.
 (d) Find all upper bounds for $\{(2, 3), (3, 2)\}$.
 (e) Find all lower bounds for $\{(2, 3), (3, 2)\}$.
 (f) Find the least upper bound for $\{(2, 3), (3, 2)\}$.
 (g) Find the greatest lower bound for $\{(2, 3), (3, 2)\}$.
 (h) Find the least element.
 (i) Find the greatest element.
 (j) Is this a lattice?
14. Carefully prove the following relations are partial orders.
- (a) Recall the product order: Let (A_1, \preceq_1) and (A_2, \preceq_2) be posets. Define the relation \preceq on $A_1 \times A_2$ by $(a_1, a_2) \preceq (b_1, b_2)$ if and only if $a_1 \preceq_1 b_1$ and $a_2 \preceq_2 b_2$. Prove the product order is a partial order.
 (b) Let (B, \preceq_B) be a poset and let A be a set. Define the set $FUN(A, B)$ to be the set of all functions with domain A and codomain B . Prove the relation \preceq_F on $FUN(A, B)$ defined by $f \preceq_F g$ iff $f(t) \preceq_B g(t) \forall t \in A$ is a partial order.
 (c) Let (A, \preceq_A) and (B, \preceq_B) be posets and recall the lexicographic order, \preceq_L on $A \times B$ is defined by

$$(a, b) \preceq_L (c, d) \Leftrightarrow [(a \preceq_A c) \wedge ((a = c) \rightarrow (b \preceq_B d))].$$

Prove the lexicographic order on $A \times B$ is a partial order.

15. Suppose R is a relation on A . Using the property that composition is associative and mathematical induction, prove that $R^n \circ R = R \circ R^n$.
16. Prove that a relation R on a set A is transitive if and only if $R^n \subseteq R$ for all positive integers n .
17. Suppose (A, \preceq) is a poset such that every nonempty subset of A has a least element. Prove that \preceq is a total ordering on A .
18. Prove: Suppose (A, \preceq) is a finite nonempty poset. Then A has a minimal element.