Homework 2
50 points

Context

Define a function f to be *admissable* iff it is defined on almost all of the non-negative integers and has positive real values on almost all of the domain. That is, there is an integer n_0 such that $f(n)$ is defined and $f(n) > 0$ for all integers $n > n_0$. Note that for polynomials, this condition requires that the leading coefficient is a positive number.

Asymptotic Equivalence and Tilde Notation

For admissable functions f and g, define $f \sim g$ to mean that

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1.$$

We show below that $\sim$ is an equivalence relation. The terminology used for $f \sim g$ is that f and g are *asymptotically equivalent*.

Proposition 1. $\sim$ is an equivalence relation on the set of admissable functions.

Hint. Recall you need to prove the three conditions:

Reflexive: $f \sim f$ for all f

Symmetric: $f \sim g$ implies $g \sim f$ for all f, g

Transitive: $f \sim g$ and $g \sim h$ implies $f \sim h$, for all f, g, h

Proposition 2. Suppose f and g are admissable and

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0.$$

Then $f + g \sim g$.

Proposition 3. Suppose f and g are admissable and $f \sim g$. Then $f = \Theta(g)$.

Exercise 4. Is the following inverse of Proposition 3 true? Suppose f and g are admissable and $f = \Theta(g)$. Then $f \sim Cg$ for some positive constant C. (Prove true or false)

Exercise 5. Prove true or false:

(a) $n \log n + \log n \sim n \log n$

(b) $n \log n + n \sim n \log n$

(c) $n \log n + n \log n \sim n \log n$