

### Assignment 3

#### 50 Points

**Problem 1.** Consider hash tables with collision resolved by chaining, implemented as vector-of-lists, as in `fsu::HashTable<K,D,H>`. Show that the standard traversal has runtime  $\Theta(b + n)$ , where  $b$  is the number of buckets and  $n$  is the size of the table. Use the context and notation established on the next page. (Hint: use aggregate analysis.)

**Problem 2.** Consider the `Partition` data structure implementing the Union/Find disjoint sets algorithms. Let  $T$  be any tree in the forest, and denote the rank of  $T$  by  $d$  and the number of elements of the set represented by  $T$  by  $k$ . Show that  $d \leq \log_2 k$ . (Hint: Use mathematical induction on  $d$ . For the inductive step, examine the tree of rank  $d$  with the fewest number of nodes.)

**Problem 3.** Consider the family of rectangular mazes described in Disjoint Sets Appendix: Maze Technology.

- (a) Devise an algorithm that translates a 2-D maze of square cells into a graph whose characteristics reflect all properties of the maze. For example, a path in the graph would correspond to a path in the maze. (We'll refer to this translation as an *isomorphism*.)
- (b) Describe in more general terms how the isomorphism would generalize to 2-D mazes of cells of other shapes, such as hexagonal, or variable shape as long as the shapes are polygons. (E.g., any tile floor would do.)
- (c) Based on the technology for 2-D mazes of square cells, invent maze technology for describing 3-D mazes of cubical cells. How would the isomorphism generalize to this case?

```

// standard traversal of HashTable t:
for (HashTable::Iterator i = t.Begin(); i != t.End(); ++i)

// HashTable and HashTableIterator context:
class HashTable
{
public:
    typedef HashTableIterator Iterator;
    Iterator Begin();
    Iterator End(); // known to be constant time
    ...
private:
    Vector<List> v; // vector of lists (bucket vector)
};

class HashTableIterator
{
public:
    typedef HashTableIterator Iterator;
    Iterator& operator++();
    ...
private:
    unsigned vi; // vector index
    ListIterator li; // bucket iterator
};

// algorithms used in traversal:
HashTableIterator HashTable::Begin()
{
    Iterator i;
    i.vi = 0; // start at 0th bucket
    while (v[i.vi].Empty()) // while bucket is empty
        ++i.vi; // go to next bucket
    i.li = v[i.vi].Begin(); // start at beginning of this bucket
    return i; // NOTE: Begin() == End() for an empty bucket
}

HashTableIterator& HashTableIterator::operator++()
{
    ++i.li; // go to next item in bucket
    if (i.li == v[i.vi].End()) // if at end of bucket
    {
        do
            ++i.vi; // go to next bucket
        while (v[i.vi].Empty()); // until bucket is not empty
        i.li = v[i.vi].Begin(); // start at beginning of this bucket
    }
    return *this; // NOTE: Begin() == End() for an empty bucket
}

```