\[ u = n + S \] and/or when \( \forall \cdot \)

The new definition must be consistent with the definitions

\[ S \not\ni n \]

We have to redefine what it means to solve a system when

\[ \exists \cdot \]

If \( \forall \cdot \) then there cannot exist a \( c \in \mathbb{R} \) such that \( n = A c \).

If \( \forall \cdot \) then there exists a unique \( c \in \mathbb{R} \) such that \( n = A c \).

If \( \forall \cdot \) then we do not have the notion of \( A^{-1} \).

\[ S = (A) \mathcal{A} = [a_1, \ldots, a_m] \text{span} \]

The columns therefore form a basis for a space.

Assume that \( A \in \mathbb{R}^{n \times n} \), \( \forall \cdot \), has linearly independent columns.

Subspaces, Bases, Orthogonality and Optimization.
basis.

the unique coefficient vector associated with \( \mathbf{v} \) relative to this new

The columns of \( \mathbf{A} \) also form a basis and we can easily determine

consider the matrix \( \mathbf{A} \).

Let \( \mathbf{W} \in \mathbb{F}^{m \times (n-r)} \) be a nonsingular matrix, i.e., \( \mathbf{W}^{-1} \) exists. Now

\[ \mathbf{W} \mathbf{v} = \mathbf{w} \in \mathbb{F}^m. \]

Suppose \( \mathbf{v} \in \mathbb{F}^n \) such that

there exists a unique \( \mathbf{c} \in \mathbb{F}^r \) such that

\[ \mathbf{W}_\mathcal{S} = \mathbf{A} \mathcal{U} \] for \( \mathcal{S} \). In fact there are

We have that the columns of \( \mathbf{A} \) form a basis for \( \mathcal{S} \).
\( \mathbf{B} \) is full rank its columns form a basis for \( \mathcal{R}(\mathbf{B}) \).

Since \( \mathbf{B} \) is full rank, therefore, \( \mathbf{B} \) is of full rank.

Therefore, \( \mathbf{W} \) is nonsingular (full rank) and therefore no such assumption is made.

But by the assumption of full rank (linearly independent columns), the following holds:

\[
0 = \mathbf{x} \mathbf{W} \quad \text{then} \quad 0 \neq \mathbf{x} = \mathbf{x} \mathbf{W} \mathbf{A} = \mathbf{B} \mathbf{x}
\]

Suppose \( \mathbf{A} \) is full rank.
\begin{align*}
\text{LEMMA: } & \forall \in (B) \forall \in = (qW) \forall = \forall c = w \\
&\text{and } \forall \in (B) \forall = (qW) \forall = \forall c = w \\
&\text{We have written uniquely as } W = q \forall c = \text{by taking } W = q \forall c = \text{since } W \in \mathcal{H} \times \mathcal{Y}. \text{ But any } c \in \mathcal{Y} \text{ can be assumed } c \in \mathcal{Y}. \text{ Therefore, } \forall \in (B) \forall = (qW) \forall = \forall c = w \text{ where } \\
&\text{we therefore have } \forall \in (B) \forall = (qW) \forall = \forall c = w \text{ such that } \\
&\text{then there exists a unique } c \in \mathcal{Y} \text{ such that } \\
&\forall \in (B) \forall = \mathcal{S} = q(\forall) (B) \forall = \mathcal{S} = q(\forall) (B)
\end{align*}
transformation $W \in \mathbb{R}^{m \times n}$.

It can also be shown that any two matrices whose columns form bases of the same space must be related by a nonsingular change of the range space only if the particular basis

non-singular transformation $W \in \mathbb{R}^{m \times n}$ applied to $A$ does not

So if you are given a basis in the form of a matrix $A \in \mathbb{R}^{m \times n}$, any
\[ b = a \perp \mathcal{O} \]
\[ b^\perp_\mathcal{I} = a \perp \mathcal{O} \]
\[ b(\mathcal{O} \perp \mathcal{O}) = a \perp \mathcal{O} \]
\[ b\mathcal{O} \perp \mathcal{O} = a \perp \mathcal{O} \]
\[ b\mathcal{O} = a \]

We know \( b \in \mathcal{Y} \), \( \mathcal{Y} \) exists such that \( b\mathcal{O} = a \). How do we find it?

\[(\mathcal{O} \mathcal{Y} \mathcal{U}) \subset \mathcal{Y} \mathcal{U} \mathcal{Y} \in \mathcal{Y} \].

To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square.

Determining coefficients is simple in this case even though \( \mathcal{O} \mathcal{Y} \mathcal{U} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square. To see this consider \( \mathcal{O} \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \) is not square.

Now suppose we have \( \mathcal{Y} \mathcal{U} \in \mathcal{Y} \mathcal{U} \mathcal{Y} \).
column of \( A \) with \( v \). The coefficient of \( v \) is just the inner product of the \( \nu \), the

\[ (A\nu) \]

that \( v \in \mathbb{R} \) and \( A \) is an orthogonal matrix.

So we can easily solve this rectangular equation if we know
\[
\begin{pmatrix}
\varepsilon_A \\
\tau_A
\end{pmatrix}
= b
\]

\[
b = a \begin{pmatrix}
0 & \cdots & 1 & 0 \\
0 & \cdots & 0 & 1
\end{pmatrix}
\]

\[
b = a L \mathcal{O}
\]

\[
b \mathcal{O} = a
\]

\[\mathcal{O} \nu \in \mathcal{V} \]

and also \( \nu \in \mathcal{V} \). Suppose \( \nu \in \mathcal{V} \).

The standard basis shows this trivially. Suppose \( \nu \in \mathcal{V} \).
so it is positive definite.

\[
0 < \frac{1}{2}\|m\| = (m)(\mathcal{L}m) = (x\mathcal{V})(\mathcal{L}x\mathcal{V}) = x(\mathcal{V}\mathcal{L}x) = xN\mathcal{L}x
\]

and for \(0 \neq x\) for \(x\mathcal{N}\mathcal{L}x\)

**NOTE:** Consider \(N\). \(\mathcal{V}\mathcal{L}x = N\). It is obviously symmetric. Form a basis for \(\mathcal{V}\mathcal{L}\). Suppose \(\mathcal{A} \in \mathbb{R}^{m \times n}, U \geq u\), where \(\mathcal{A}\) is full rank. The columns of \(\mathcal{A}\) exist. In fact, an orthonormal basis must always exist.

Our earlier results say that if \(O \in \mathcal{V}\mathcal{L}\), there exists there must be a nonsingular transformation relating it to all other bases of the range space.
of are an orthonormal basis.

\[ \mathcal{V} = (\mathcal{O}) \mathcal{V} = (\mathcal{O}) \mathcal{V} \]

Therefore \( t^{-1} \mathcal{V}_{H} = \mathcal{O} \).

**NOTE:** Follows:

\[ t^{-1} \mathcal{V}_{H} = \mathcal{O} \]

Define \( \mathcal{V}_{H} = \mathcal{O} \).

Exist.

upper triangular with positive diagonal elements must therefore

**NOTE:** A Cholesky factorization.
Since we know that $\mathcal{O} \neq q$, we only know that $b \in \mathcal{O} \in S$. Since $\mathcal{O} \neq q$, we know that $\mathcal{O} = c$

\[
q \downarrow \mathcal{O} = c
\]

We proceed as before and form

\[
S = (\mathcal{O})^2 \neq q \text{ but } q \in \mathcal{O} \text{ and } q \in (\mathcal{O})^2 \text{ is full rank and}
\]

First we suppose we have $u \in \mathcal{O}$, $u \in \mathcal{O}$, and $q \in \mathcal{O}$, and $q \in (\mathcal{O})^2$ is full rank and

\[
q = x^2
\]

solve

So what? This allows us to explore the question of how well can we
\[ q(L \tilde{\mathcal{O}} \tilde{\mathcal{O}} - I) = \]
\[ q_L \tilde{\mathcal{O}} \tilde{\mathcal{O}} - q = \]
\[ c \tilde{\mathcal{O}} - q = \]
\[ b - q = \mu \]

**NOTE:**

Can we say anything about it?

\[ c \tilde{\mathcal{O}} - q = b - q \]

Consider the residual.
\[ 0 = \]
\[ q(u_0) = \]
\[ q(\hat{\mathcal{O}} - \hat{\mathcal{O}}) = \]
\[ q(u I \hat{\mathcal{O}} - \hat{\mathcal{O}}) = \]
\[ q(\hat{\mathcal{O}} \hat{\mathcal{O}} I \hat{\mathcal{O}} - \hat{\mathcal{O}}) = \]
\[ q(\hat{\mathcal{O}} \hat{\mathcal{O}} I \hat{\mathcal{O}} - I) = u \hat{\mathcal{O}} \]

*NOTE*
\[ \top b + b = q \]

into two vectors that are unique and orthogonal, i.e., we have \( \top b = \lambda \) is orthogonal to \( S \) and we have split \( q \).
Finally note that if $v = v_1 + v_2$ where $v_1^Tv_2 = 0$ then

$$\frac{||v||^2}{||v_1||^2 + ||v_2||^2} = \frac{v^Tv}{v_1^Tv_1 + v_2^Tv_2 + 2v_1^Tv_2}$$
The norm of the residual is the vector $x \in \mathcal{Y}$ that minimizes over all vectors in $\mathcal{Y}$ the two

$$
q = \text{solution to the system}
$$

Assume that $A \in \mathbb{R}^{m \times n}$, is of full rank and $q \in \mathcal{Y}$, is square nonsingular matrices.

We now have the machinery to define the solution of problems

Linear Least Squares Problems
do we have a nonzero residual.

\[ S = (\forall) \forall \neq q \text{ when } \text{only when } u = \text{ when } \text{(1.4)} \]

\text{NOTE: If } q \in q \text{ then the residual can be made 0.}
\[
\frac{\zeta}{\zeta} \| x^A - \zeta q \| + \frac{\zeta}{\zeta} \| q \| = \frac{\zeta}{\zeta} \| x^A - q + q_x \| = \frac{\zeta}{\zeta} \| x^A - q \|
\]

We have

Also, by definition, \( A^\perp \notin S \) and therefore \( q^\perp \notin A^\perp \). Where \( q^1 \in S^T \) and \( q^2 \in S \).

\[ q^2 + q^1 = q \]

By our previous results we know that
and multiplication with $A$, left over is in a space, $S^\perp$, that we cannot reach with values of $x$.

What is the problem is solved by working in the range space $S$. We have

\[
0 = \mathbf{J} \mathbf{V}
\]

\[
\mathbf{S}^\perp \ni \mathbf{J}
\]

\[
\|\mathbf{J} \mathbf{q}\| = \|\mathbf{J}\|
\]

\[
\|\mathbf{J} \mathbf{q}\| = \|\mathbf{J} \mathbf{V} - \mathbf{q}\|
\]

and therefore we have

\[
\mathbf{J} \mathbf{q} = \mathbf{V} \mathbf{x}
\]

We know that we can find the unique $x$ that solves $\text{null} x = x$. We
The use of orthogonal factorization.

The use of producing an orthogonal basis.

The use of the normal equations.

We will explore implicitly or explicitly.

notes, i.e., orthogonality and basis construction either of A. Clearly, we must exploit the algebra derived in this set of so we wish to solve the system Ax = b, by restricting q to the range