- Update $BB^T = R^T R$ - Cholesky factorization
- Update $\mathcal{L}^T \mathcal{L} = B$ - Factorization
- Represent via $B^{[0]}$ and simple updates
- Update $B^{-1}$

- Each step to get $O(m^2)$ per step.

Could exploit the fact that is changed by a rank 1 matrix on

- Could factor new on every step requiring $O(m^3)$.  

So we have three systems with $B$ as the matrix of coefficients.
is upper triangular with last column $P$.

\[ P^\phi \Phi P = I \]

with the $l$-th. Then

Now define a permutation matrix $P$ that interchanges the $m$-th row with the identity $I$ in the $l$-th column replaced by $m$.

\[
\begin{align*}
-I^{-1} & = m \in \mathbb{N} P^\phi I^{-1} \\
-I^{-1} P = P^{-1} & = \quad m \in \mathbb{N} P^\phi I^{-1} \\
-I^{-1} (P^\phi m + I) & = \quad m \in \mathbb{N} P^\phi I^{-1} \\
-I^{-1} ((l \in \mathbb{N} P - m + I) P) & = \quad m \in \mathbb{N} P^\phi I^{-1} \\
-I^{-1} (l \in \mathbb{N} P - l \in \mathbb{N} P^\phi N - P) P + P & = \quad m \in \mathbb{N} P^\phi I^{-1} \\
-I^{-1} (l \in \mathbb{N} P - l \in \mathbb{N} P^\phi N + P) & = \quad m \in \mathbb{N} P^\phi I^{-1}
\end{align*}
\]

Update Inverse
\[
\begin{pmatrix}
0 & 0 & \varepsilon & 0 \\
0 & 1 & \varepsilon & 0 \\
1 & 0 & \varepsilon & 0 \\
0 & 0 & \varepsilon & 0 \\
\end{pmatrix}
= \gamma \Phi d
\]

\[
\begin{pmatrix}
0 & 0 & \varepsilon & 0 \\
0 & 1 & \varepsilon & 0 \\
1 & 0 & \varepsilon & 0 \\
0 & 0 & \varepsilon & 0 \\
\end{pmatrix}
= \gamma \Phi
\]

\[
\exists = \gamma \text{ and } \forall = \mu \text{ with }
\]
\[
\begin{pmatrix}
\varepsilon m & 0 & 0 & 0 \\
\varepsilon m & 1 & 0 & 0 \\
\varepsilon m & 0 & 1 & 0 \\
\varepsilon m & 0 & 0 & 1 \\
\end{pmatrix}
\] = \int d^3 \Phi d
So each column of $B_{1-I}$ can be created by solving a triangular system.
\[ \Phi_{i-1} \Phi \Phi \cdots \Phi_{i-1} \Phi = n \]
\[ \omega = \Phi_{i} \cap \mathcal{I} \]
\[ \omega = n^{\Phi_{i}} \Phi \cdots \Phi_{i-1} \Phi \cap \mathcal{I} \]
\[ \omega = n_{(i)} \mathcal{B} \]

So

\[ \Phi_{i} \cdots \Phi_{i-1} \Phi_{(0)} \mathcal{B} = (i) \mathcal{B} \]

Suppose \( \mathcal{I} \cap \mathcal{I} = (0) \mathcal{B} \) is known.

We know that \( \Phi_{i} \Phi_{(1-i)} \mathcal{B} = (i) \mathcal{B} \) at step \( i \).

Let \( \mathcal{B} \) be the \( m \times m \) matrix whose columns are the basis needed.

Relate to Initial \( \mathcal{B} \)
it gets too large you can compute all of \((i)B\) and factor it to start
\[ (\zeta)O^m + in + (1)O + \ldots \]

simple tridiagonal solves as discussed before.

So to solve we have an iteration followed by
\[(\mathcal{N} \mathcal{Q})(\mathcal{T} \mathcal{T}) = \]
\[\mathcal{N} (\mathcal{Q} \mathcal{T}) \mathcal{T} = \]
\[\mathcal{N} (\mathcal{L} \mathcal{N} \mathcal{x} + I) \mathcal{T} = \]
\[\mathcal{N} (\mathcal{I} - \mathcal{N} \mathcal{L} \mathcal{x} \mathcal{I} - I + I) \mathcal{T} = \]
\[\mathcal{N} (\mathcal{L} \mathcal{N} \mathcal{x} \mathcal{I} - I + \mathcal{N}) \mathcal{T} = \]
\[\mathcal{L} \mathcal{N} \mathcal{x} + \mathcal{N} \mathcal{T} = \mathcal{T} \mathcal{m} \mathcal{m} \mathcal{A} \mathcal{T} \mathcal{m} \mathcal{m} \mathcal{A} \mathcal{T} \]
\[\mathcal{N} \mathcal{T} = \mathcal{B} \]

\[\mathcal{h} = \mathcal{h} \mathcal{L} \mathcal{N} \text{ and } x = \mathcal{N} \mathcal{T} \text{ and } \mathcal{L} \mathcal{N} = \mathcal{B} \text{ Assume the \textit{factorization} of } \mathcal{B} \text{ to use in the solvers} \]

\[\text{We can update the } \mathcal{L} \mathcal{N} \text{ factorization.} \]
Note we have ignored details such as pivoting.

\( O(m^2) \) \( \): During the factorization and keep the cost \( N \) (and \( T \) and \( T \) T) and \( N \) and Murray have shown that you can also form \( \)

\[ L \bar{\xi} x + I = \Omega T \]

In fact, Golub, Gill, and Murray have shown that \( m \) operations at each step can be produced in \( O(m^2) \) operations. (The active part of the matrix is also a rank-1 update of the diagonal matrix and can be computed in \( O(m^2) \) operations.)
During each step we must solve systems of the form $\mathbf{a} = \mathbf{n}_B \mathbf{b}$ or $\mathbf{a} = \mathbf{n}_B$. Decompositions write the problem in terms of Cholesky factors and orthogonal stability problems can occur with the previous approaches. We can
So we can handle \( B^n = n \) systems.

\[
\begin{align*}
\rho &= R_n R_n^T \\
\rho^n &= n R_n B B^T \\
\rho &= n B B^T \\
R_n R_n^T &= B B^T
\end{align*}
\]

such that have an upper triangular matrix \( R \) with positive diagonal elements. We therefore have a Cholesky factorization, i.e., since \( B \) is full rank. So consider \( B B^T \in \mathbb{R}^{m \times m} \). It must be symmetric positive definite.
Updating $R$.

"or $R^{-1}$ with one of updating the factorization of $BB^T$, i.e.,

So we have altered the problem from updating a factorization of $B$

\[
B^{\perp n} = n \\
\alpha = \eta R \perp n \\
\alpha = (\eta \perp n - B)B^T \perp n \\
\alpha = \eta n
\]

We can also handle $Bn = \eta$.\n
Consider

We can handle the update of $R$ by relating it to a QR factorization.
\[
\begin{pmatrix}
L_0 \\
\mathcal{H}
\end{pmatrix}
= 
\begin{pmatrix}
L_m \\
\mathcal{H}
\end{pmatrix}
\text{O}
\]

such that

\((\mathcal{I} + \mathcal{W}) \times (\mathcal{I} + \mathcal{W})\mathcal{H} \in \mathcal{O}\).

Now suppose we define an orthogonal matrix \(\mathcal{H} \in \mathcal{O}\).

We can write this as

\[
L^{\text{nn}} + \mathcal{H} \mathcal{H} = \begin{pmatrix}
L_m \\
\mathcal{H}
\end{pmatrix}
\begin{pmatrix}
\mathcal{I} & \mathcal{H} \\
\mathcal{H} & \mathcal{R}
\end{pmatrix}
\]

Consider the inclusion of the first term to update \(\mathcal{H}\), i.e.
\[ \mathcal{L} = \mathcal{H} \]

\[ \mathcal{L}^{n+m} + \mathcal{H} \mathcal{L} \mathcal{H} = \]

\[ \left( \begin{array}{c} \mathcal{L}^n \\ \mathcal{H} \end{array} \right) \left( \begin{array}{c} m \\ \mathcal{L} \mathcal{H} \end{array} \right) = \]

\[ \left( \begin{array}{c} \mathcal{L}^n \\ \mathcal{H} \end{array} \right) \varprod \left( \begin{array}{c} m \\ \mathcal{L} \mathcal{H} \end{array} \right) = \mathcal{H} \mathcal{L} \mathcal{H} \]

We then have
Use two rows to define a $2 \times 2$ orthogonal matrix and embed in a

using an orthogonal transformation.

\[
\begin{pmatrix}
L^m \\
R
\end{pmatrix}
\]

So we can update $R$ by eliminating $m$. 

continue until entire row 6 is 0s

rotate rows 2 and 6

rotate rows 1 and 6
alternative to orthogonality is defined.

The problem is called **downdating** a factorization. To solve it an

\[ I = \emptyset_L \emptyset \]

The approach above will **always generate a** sign since

\[ L^L L = m_{\alpha u} R^{m_{\alpha u}} R \]

We cannot use exactly the same approach to handle the update
\[
\left( \begin{array}{c}
\mathcal{L}_0 \\
\mathcal{L}_u
\end{array} \right)_{m \in \mathcal{H}} = \left( \begin{array}{c}
\mathcal{L}_u \\
\mathcal{L}_\Omega
\end{array} \right)_{\mathcal{O}}
\]

Choose a hyperbolic \( \mathcal{O} \) with \( \mathcal{O} = I \) such that

\[
\left( \begin{array}{cc}
\mathcal{L}_u & 0 \\
0 & \mathcal{L}_I
\end{array} \right) = \mathcal{O} \left( \begin{array}{cc}
\mathcal{L}_u & 0 \\
0 & \mathcal{L}_I
\end{array} \right) \mathcal{O}
\]

Hyperbolic transformations are such that
\[
\begin{pmatrix}
0 \\
\theta
\end{pmatrix} = \begin{pmatrix}
\cosh \\
\sinh
\end{pmatrix} \begin{pmatrix}
c & s-
\end{pmatrix}
\]

can construct such that \( H \) can be built from hyperbolic rotations. Consider a \( 2 \times 2 \) matrix

\[
\begin{pmatrix}
c & s-
\end{pmatrix}
\begin{pmatrix}
c & s-
\end{pmatrix} = H
\]

where \( c = \cosh \), then we have \( H S_j H = S \).

Let \( \theta \) be some angle.
\[(\theta)\sin s = 1 + u \Theta H_{d\Theta} = d\Theta H^{1+u\Theta} \]

\[(\theta)\cosh = 1 + u \Theta H^{1+u\Theta} = d\Theta H_{d\Theta} \]

Such a matrix is the identity everywhere except for an element as desired above. This can be embedded in an \(u + 1\) hyperbolic matrix that has the signature matrix with a single \(-1\) in the lower right corner.
Stewart.

More detailed and advanced error analyses are due to
Brent and Van Loan in the standard approach
The method of mixed rotations analyzed by Bojanczyk,
three basic approaches
Stewart 1998 has a discussion that includes pointers to the
Golub and Van Loan discuss the simplest version

There are multiple approaches

Complexity

\( (u^0) \cdot \text{Toeplicitz factorizations with } O(n^2) \)

They are used to develop fast methods for structured dense

They have some stability problems and one must be careful.

Hyberbolic rotations and the related Hyperbolic reflectors can