\[ q_\perp(\mathcal{H} + \mathcal{V}) = x \]

\[ q_\perp \mathcal{V} = x \]

then the columns of \( \mathcal{V} \) are linearly independent and if

\[ \frac{\| \mathcal{V} \|}{\| \mathcal{H} \|} > \frac{1}{2} \]

If projections of \( \mathcal{H} \) onto \( \mathcal{V} \) and \( q \) onto \( \mathcal{H} \) and \( q \) be projections of \( \mathcal{V} \) onto \( \mathcal{H} \) and \( q \) let \( \mathcal{H} \), let \( \mathcal{V} \), and let \( q \) be independent columns, let \( \mathcal{H} \) \( \mathcal{V} \) \( q \) and let \( q \) have linearly

**THEOREM** (Stewart 1973) Let \( \forall \mathcal{H} \in \mathbb{R}^{m \times n} \text{ and } q \in \mathbb{R}_{}^{n} \) have linearly

Perturbation Theorem for Least Squares
\[(A_{F}A)^{\perp} = \mathcal{N}\]

where \(\mathcal{N}\) and it can be shown that

\[
\frac{\|A\|}{\|A\|_{2}} + \frac{\|q_{1}\|}{\|A\|_{2}} + \frac{\|y\|_{2}}{\|A\|_{2}} + \frac{\|v\|}{\|A\|_{2}} \geq \frac{\|x\|}{\|x - x\|}
\]

**THEOREM**
\[ (\forall) \mathcal{N} \]

weighted by the size of the projection of \( q \) that is orthogonal to

The second term has a larger expansion factor, \( \mathcal{N} \), but is

by the condition number.

that the part of the relative error that is in \( (\forall) \mathcal{N} \) is expanded

The first term is like our bound on solving systems and says

The third term can usually be ignored due to the square.
The Householder method is very stable when computing the transformed problem (and when used to compute projections or an orthogonal basis).

CGS and MGS are unreliable as producers of an orthogonal basis except for well-conditioned problems unless reorthogonalization is used which doubles the operation count.

MGS is as stable as Householder for the least squares problem even though the computed $Q$ may not be orthogonal.

The normal equations can have difficulties from the backward error point of view (the perturbation that one must use is proportional to the condition number). However, for moderately well-conditioned problems in practice it is often accurate enough. Note the perturbation effect to the normal equations themselves is proportional to $\kappa^2$. 
perform a rank-revealing QR factorization. Column pivoting is also needed when you are attempting to

added to Householder, MGS and CGS. For less well-conditioned problems, column pivoting can be
to be used.

other orthogonal transformation-based triangularization (or
In general, for black box library routines the Householder (or
for the normal equations (by a factor of 2).

The smallest operation count for the least squares problem is

•