Session: File System and Commands
Topic: Numbering Systems

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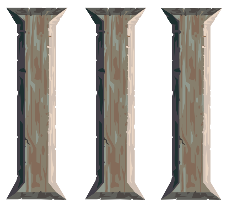
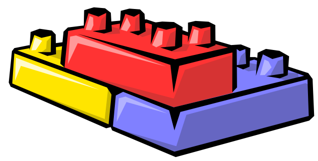
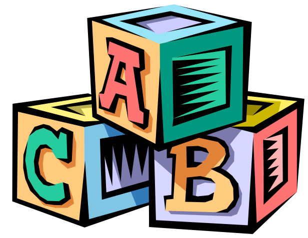
Number vs. Numeral

- What is the difference between a number and a numeral?

Numeral (Base 10 - Decimal)	Number (A Quantity)



- A "Numeral" is a written representation of the concept of a "Number" of some objects

Analogy

- Coins can be thought of a symbolic representation of some "Number" of cash

- We make change for a given "Number" ("Quantity") using some combination of Coins with various values
- How do we use "Numerals"?

Decimal Numbering System (Base 10)

Column Values	1000	100	10	1	
Digits	5	0	2	3	
Total	5 * 1000	0 * 100	2 * 10	3 * 1	Quantity
	5000	+ 0	+ 20	+ 3	= $(5023)_Q$

- Each column has a value (ones, tens, hundreds)
- Actual column value is base to an exponent (hence base-10)

Column Values	1000	100	10	1
	= 10^3	= 10^2	= 10^1	= 10^0

General Numbering System (Base X)

Actual Values	X^3	X^2	X^1	X^0
Possible Digits	0 to (X-1)	0 to (X-1)	0 to (X-1)	0 to (X-1)

- Any column can only count up to (X - 1), because one more would roll over to the value of the next column (to left)

Common Bases

Base	Common Name	Representation	Digits
10	Decimal	$(N)_{10}$	0-9
2	Binary	$(N)_2$	0-1
8	Octal	$(N)_8$	0-7
16	Hexadecimal	$(N)_{16}$, 0xN	0-9, A-F

Binary

- The numbering system using only zeroes and ones is called "Binary"
- A single binary digit ("0" or "1") is a "bit"
- Can use varying number of bits to create different patterns to represent larger numbers of different "things" (letters, etc.)

Binary Numbering System

- Binary is the numbering system used in computers
- Here bits are used to represent actual numeric values
- For any pattern of bits, each position is given a value and a "1" means you include the value while a "0" means you do not

Column Values	8	4	2	1	
Digits	1	0	1	1	
Total	1 * 8	0 * 4	1 * 2	1 * 1	Quantity
	8	+ 4	+ 2	+ 1	= $(15)_Q$

- The right-most digit in a binary number is always worth "1"
- Each other column has a value that is two (2) times the column to the left
- Actual column value is the base "2" to an exponent
- Hence "binary" is also called "base-2"

Column Values	8	4	2	1
	= 2^3	= 4^2	= 2^1	= 1^0

What quantity of objects is represented by the binary numeral:

- $(1)_2$
- $(101)_2$
- $(00000001001)_2$
- $(11001)_2$

Hexadecimal

- Hexadecimal (base-16) is another numbering system used in computers
- Each single digit position can have up to 16 different values (compared to 10 or 2)
- Each single digit is represented by (0-9) and then (A-F)
- The right-most digit in a hexadecimal number is always worth "1"
- Each other column has a value that is sixteen (16) times the column to the left

Column Values	4096	256	16	1	
Digits	0	0	2	A	
Total	0 * 4096	0 * 256	2 * 16	10 * 1	Quantity
	0	+ 0	+ 32	+ 10	= $(42)_Q$

- Each Hexadecimal digit is equivalent to the value of four (4) binary digits, and you can easily convert between the two
- You must have exactly four (4) binary digits for one (1) hexadecimal digit

					Quantity
Hex	0	0	2	A	= $(42)_Q$
Binary	0000	0000	0010	1010	= $(42)_Q$

- Hexadecimal numerals are often indicated using "0x"

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0xb3fd
0xFF
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What quantity of objects is represented by the hexadecimal numeral:

- 0x4
- 0xB
- 0x001F

What is the binary equivalent to:

- 0x 0032 B9A5

Conversion

- First convert numeral to quantity (number) by adding up amounts of column values

Column Values	1000	100	10	1	
$(13)_{10}$	0	0	1	3	
Total	0	0	1 * 10	3 * 1	Quantity
			10	+ 3	= $(13)_Q$

- Then convert quantity to target base by selecting amounts from each column, starting with largest possible
- Similar to making change in a very unusual coin system

Column Values	512	64	8	1	Base-8
$(13)_Q$	0	0	1	5	= $(15)_8$
Total	0	0	1 * 8	5 * 1	
Remain			= 5	= 0	

Tips

- Decimal numerals are essentially equivalent to the quantity (because we think in decimal)
- Bases that are multiples of each other have direct correspondence between digits, so can convert directly between bases

Bases

base	base^{6}	base^{6}	base^{5}	base^{4}	base^{3}	base^{2}	base^{1}	base^{0}	base^{-1}	base^{-2}	base^{-3}
2	128	64	32	16	8	4	2	1	0.5	0.25	0.125
8	-	-	32768	4096	512	64	8	1	0.125	0.015625	-
16				65536	4096	256	16	1	0.0625	-	-

Conversions

Quantity (Value)	Numerals				
	base-10	base-2	base-8	base-2	base-16
0	0	000	0	0000	0
1	1	001	1	0001	1
2	2	010	2	0010	2
3	3	011	3	0011	3
4	4	100	4	0100	4
5	5	101	5	0101	5
6	6	110	6	0110	6
7	7	111	7	0111	7
8	8	1 000	10	1000	8
9	9	1 001	11	1001	9
10	10	1 010	12	1010	A
11	11	1 011	13	1011	B
12	12	1 100	14	1100	C
13	13	1 101	15	1101	D
14	14	1 110	16	1110	E
15	15	1 111	17	1111	F

Examples

Given the numeral $(27)_{10}$, what is the equivalent numeral in

- Binary
- Base-16

Given the numeral $(34)_{8}$, what is the equivalent numeral in

- Base-10
- Base-2, which is then equivalent to what Decimal numeral?
- Base-2, calculated in less than 2 seconds

What is the binary equivalent to:

- $(F9C5)_{16}$

Mathematically

$(27)_{10}$ $= (?)_{2}$

$= (?)_{16}$

$(34)_{8}$ $= (?)_{10}$

$= (?)_{2}$ $= (?)_{10}$

$= (??)_{2}$

$(F9C5)_{16}$ $= (?)_{2}$