COP5025 Spring 2000 − Final Exam (Chs. 8+9+Prolog)

Name: _____ (Please print)

You can put the answers on these sheets. Use additional sheets when necessary. Always show how you derived your answers (this is required for full credit and helpful for partial credit). You can collect 100 points in total for this exam. A bonus question is included for an additional 15 points. **This exam is open book and open notes.**

1. Consider the language of simple expressions (Section 8.6). What functions do the following expressions denote? (15 points)

   (a) $x + y$

   (b) **let** $x = 1$ **in** $x + y$ **end**

   (c) **let** $y = y$ **in let** $x = y$ **in** $x - y$ **end end**

2. Consider the program

   program(x); **let** $y = x + 1$ **in** y/x **end end**.

   written in the language of simple expressions with error values. Given the following
   input values, what will be the output values using the semantic definitions **given on
   slides 96 and 97**? You do **not** need to show the derivation. Give the value returned
   by the program for each case. (10 points)

   (a) input $= 1$

   (b) input $= 0$

   (c) input $= 2$

   (d) input $= -1$

3. Consider the C conditional-or operation || with the usual semantics:

   $(x \mathbin{||} y) = 0$ if $x = 0$ and $y = 0$
   $(x \mathbin{||} y) = 1$ if $x = 0$ and $y \neq 0$
   $(x \mathbin{||} y) = 1$ if $x \neq 0$

   Give a denotational semantics description of the C conditional-or operator with the
   usual behavior. (15 points) You may assume that the language of simple expressions
   is used (Section 8.6) with an additional grammar production of the form

   $E \rightarrow E \mathbin{||} E$

4. The following Prolog denotational semantics describes the possible behavior of the subtraction operator `-` in the programming language with state:

```
e(E1 - E2, R, S, (N, S1)) :-
  e(E2, R, S, (N2, S2)),
  e(E1, R, S2, (N1, S1)),
  N is N1 - N2.
```

What are the values of the following expressions, assuming initially `a=0`? (10 points)

(a) `a-1`

(b) `(a:=2)-a`

(c) `a-(a:=2)`

(d) `a-((a:=3)-1)`

5. Which of the following Hoare triples are valid? (Explain and/or show derivations) (10 points)

(a) $\{x > y\} \quad x := x - y \quad \{x > 0\}$

(b) $\{x > 0\} \quad$ if $x = 0$ then $x = y$ else $y = x \quad \{y > 0\}$

(c) $\{x < 0\} \quad$ **while** TRUE **do** $x := x + 1 \quad \{x = 0\}$

(d) $\{x = 1 \,\&\, y = 2\} \quad$ skip $\quad \{y = 2\}$

6. Find a counterexample in the simple while language to the following axiom proposed for the assignment statement:

$$\{P\} \quad V := T \quad \{P \,\&\, V = T\}$$

That is, can you find an assignment that results in an invalid triple? (10 points)

7. Find the weakest precondition of the following program, given that the postcondition is $a \leq b$. (15 points)

```
if a>b then
    a := a+b
    b := a-b
    a := a-b
else
    skip
```

8. Prove that the following program computes $s = \sum_{i=0}^{n-1} a[i]$ assuming that the loop invariant is $s = \sum_{i=0}^{j-1} a[i] \& j \geq 0$. Also find the weakest precondition of this algorithm. (15 points).

```
j := 0
s := 0
while j≠n do
        s := s + a[j]
        j := j + 1
end
```

9. **bonus question** (15 points)

Prove that the following program is partially correct:

$\{TRUE\}$
```
if a>=b then
    call swap(a, b)
else
    skip
```
$\{a = n \ \& \ b = m \ \& \ n \leq m\}$

which uses the swap procedure defined by

```
proc swap(inout x, inout y);
```
$\qquad \{x = m \ \& \ y = n\}$
```
        (x, y) := (y, x)
```
$\qquad \{x = n \ \& \ y = m\}$