

2. Functional Programming

Overview

- What is functional programming?
- Historical origins of functional programming
- Functional programming today
- Concepts of functional programming
- A crash course on programming in Scheme

Note: Study Chapter 11 Sections 11.1 to 11.2, except Sections 11.2.2, 11.2.4, and 11.2.5.

What is Functional Programming?

- Functional programming is a declarative programming paradigm
 - Computation is more implicit and functional call is the only form of explicit control
- But: imperative programming languages are more widely used
 - Integrated software development environments for procedural and object oriented programming languages are "industrial strength" and often include extensive libraries
- Many (commercial) applications exist for functional programming:
 - Symbolic data manipulation
 - Natural language processing
 - Artificial intelligence
 - Automatic theorem proving and computer algebra
 - Algorithmic optimization of programs written in pure functional languages

Why Functional Programming in This Course?

- A functional language will be used to illustrate a diversity of the programming language concepts discussed in this course
- Functional programming languages are
 - Compiled and/or interpreted
 - Have simple syntax
 - Use *garbage collection* for memory management
 - Are *statically scoped* or *dynamically scoped*
 - Use *higher-order functions* and *subroutine closures*
 - Use *first-class function values*
 - Depend heavily on *polymorphism*
 - Employ *recursion* for repetitive execution
 - Programs have no *side effects* and all expressions are *referentially transparent*

Origin of Functional Programming

- *Church's thesis*:
 - All *models of computation* are equally powerful and can compute any function
- Turing's model of computation: *Turing machine*
 - Reading/writing of values on an infinite tape by a finite state machine
- Church's model of computation: *lambda calculus*
 - This inspired functional programming as a *concrete implementation* of lambda calculus
- *Computability theory*
 - A program can be viewed as a *constructive proof* that some mathematical object with a desired property exists
 - A function is a *mapping* from inputs to output objects and computes output objects from appropriate inputs
 - For example, the proposition that every pair of nonnegative integers (the inputs) has a greatest common divisor (the output object) has a constructive proof implemented by Euclid's algorithm written as a "function"

$$\text{gcd}(a,b) = \begin{cases} a & \text{if } a = b \\ \text{gcd}(a-b,b) & \text{if } a > b \\ \text{gcd}(a,b-a) & \text{if } b > a \end{cases}$$

Concepts of Functional Programming

- **Functional programming** defines the outputs of a program as mathematical function of the inputs with no notion of internal state (no side effects)
 - A *pure* function can always be counted on to return the same results for the same input parameters
 - No assignments: dangling and/or uninitialized pointer references do not occur
 - Example pure functional programming languages: Miranda, Haskell, and Sisal
- Non-pure functional programming languages include imperative features with side effects that affect global state (e.g. through destructive assignments to global variables)
 - Example: Lisp, Scheme, and ML
- Useful features are found in functional languages that are often missing in *imperative* languages:
 - *First-class function values*: the ability of functions to return *newly constructed* functions
 - *Higher-order functions*: functions that take other functions as input parameters or return functions
 - *Polymorphism*: the ability to write functions that operate on more than one type of data
 - *Aggregate constructs for constructing structured objects*: the ability to specify a structured object in-line, e.g. a complete list or record value
 - *Garbage collection*

Lisp

- Lisp (LISt Processing language) was the original functional language
- Lisp and dialects are still the most widely used
- Simple and elegant design of Lisp:
 - *Homogeneity of programs and data*: a Lisp program is a list and can be manipulated in Lisp as a list
 - *Self-definition*: a Lisp interpreter can be written in Lisp
 - *Interactive*: interaction with user through "read-eval-print" loop

A Crash Course on Scheme

- Scheme is a popular Lisp dialect
- Lisp and Scheme adopt *Cambridge Polish notation* for expressions:
 - An expression is an *atom*, e.g. a number, string, or identifier name
 - An expression is a *list* whose first element is the function name (or operator) followed by the arguments which are expressions:
(*function arg1 arg2 arg3 ...*)
- The "Read-eval-print" loop provides user interaction: an expression is read, evaluated by evaluating the arguments first and then the function/operator is called after which the result is printed
 - Input: 9
 - Output: 9
 - Input: (+ 3 4)
 - Output: 7
 - Input: (+ (* 2 3) 1)
 - Output: 7
- User can load a program from a file with the `load` function
 - (`load "my_scheme_program"`)
 - The file name should use the `.scm` extension

Note: You can run the Scheme interpreter and try the examples in these notes by executing the `scheme` command on the linprog stack (ssh linprog). To exit Scheme, type `(exit)`. You can download an example Scheme program "Eliza". More information on Scheme can be found at <http://www.swiss.ai.mit.edu/projects/scheme>

Scheme Data Structures

- The only data structures in Lisp and Scheme are *atoms* and *lists*
- Atoms are:
 - Numbers, e.g. 7
 - Strings, e.g. "abc"
 - Identifier names (variables), e.g. x
 - Boolean values true #t and false #f
 - Symbols which are quoted identifiers which will not be evaluated, e.g. 'y
 - Input: a
 - Output: *Error: unbound variable a*
 - Input: 'a
 - Output: a
- Lists:

To distinguish list data structures from expressions that are written as lists, a quote (') is used to quote the list:

```
'(elt1 elt2 elt3 ...)
```

 - Input: '(3 4 5)
 - Output: (3 4 5)
 - Input: '(a 6 (x y) "s")
 - Output: (a 6 (x y) "s")
 - Input: '(a (+ 3 4))
 - Output: (a (+ 3 4))
 - Input: '()
 - Output: ()
- Note: the empty list () is also identical to false #f in Scheme

Primitive List Operations

- car returns the *head* (first element) of a list
 - Input: (car '(2 3 4))
 - Output: 2
- cdr (pronounced "coulder") returns the *tail* of a list (list without the head)
 - Input: (cdr '(2 3 4))
 - Output: (3 4)
- cons joins an element and a list to construct a new list
 - Input: (cons 2 '(3 4))
 - Output: (2 3 4)
- Examples:
 - Input: (car '(2))
 - Output: 2
 - Input: (car '())
 - Output: *Error*
 - Input: (cdr '(2 3))
 - Output: (3)
 - Input: (cdr (cdr '(2 3 4)))
 - Output: (4)
 - Input: (cdr '(2))
 - Output: ()
 - Input: (cons 2 '())
 - Output: (2)

Type Checking

- The type of an expression is determined only at run-time
- Functions need to check the types of their arguments explicitly
- Type predicate functions:
 - (boolean? x) ; is x a Boolean?
 - (char? x) ; is x a character?
 - (string? x) ; is x a string?
 - (symbol? x) ; is x a symbol?
 - (number? x) ; is x a number?
 - (list? x) ; is x a list?
 - (pair? x) ; is x a non-empty list?
 - (null? x) ; is x an empty list?

If-Then-Else

- *Special forms* resemble functions but have special evaluation rules
- A *conditional expression* in Scheme is written using the `if` special form:

```
(if condition thenexpr elseexpr)
```

 - Input: (if #t 1 2)
 - Output: 1
 - Input: (if #f 1 "a")
 - Output: "a"
 - Input: (if (string? "s") (+ 1 2) 4)
 - Output: 3
 - Input: (if (> 1 2) "yes" "no")
 - Output: "no"
- A more general if-then-else can be written using the `cond` special form:

```
(cond listofconditionvaluepairs)
```

where the *condition value pairs* is a list of (*cond value*) pairs and the condition of the last pair can be `else` to return a default value
 - Input: (cond ((< 1 2) 1) ((>= 1 2) 2))
 - Output: 1
 - Input: (cond ((< 2 1) 1) ((= 2 1) 2) (else 3))
 - Output: 3

Testing

- `eq?` tests whether its two arguments *refer* to the same object in memory
 - Input: `(eq? 'a 'a)`
 - Output: `#t`
 - Input: `(eq? '(a b) '(a b))`
 - Output: `()` (false: the lists are not stored at the same location in memory!)
- `equal?` tests whether its arguments have the same structure
 - Input: `(equal? 'a 'a)`
 - Output: `#t`
 - Input: `(equal? '(a b) '(a b))`
 - Output: `#t`
- To test numerical values, use `=`, `<>`, `>`, `<`, `>=`, `<=`, `even?`, `odd?`, `zero?`
- `member` tests membership of an element in a list and returns the rest of the list that starts with the first occurrence of the element, or returns false
 - Input: `(member 'y '("s" x 3 y z))`
 - Output: `(y z)`
 - Input: `(member 'y '(x (3 y) z))`
 - Output: `()`

Lambda Abstraction

- A Scheme *lambda abstraction* is a nameless function specified with the `lambda` special form:
`(lambda formalparameters functionbody)`
where the *formal parameters* are the function inputs and the *function body* is an expression that is the resulting value of the function
- Examples:
 - `(lambda (x) (* x x))` ; is a *squaring function*: $x \rightarrow x^2$
 - `(lambda (a b) (sqrt (+ (* a a) (* b b))))` ; is a *function*:

$$(a\ b) \rightarrow \sqrt{a^2 + b^2}$$

Lambda Application

- A lambda abstraction is *applied* by assigning the evaluated actual parameter(s) to the formal parameters and returning the evaluated function body
- The form of a function call in an expression is:
`(function arg1 arg2 arg3 ...)`
where *function* can be a lambda abstraction
- Example:
 - Input: `((lambda (x) (* x x)) 3)`
 - Output: `9`
 - That is, `x=3` in `(* x x)` which evaluates to `9`

Defining Global Functions in Scheme

- A function is globally defined using the `define` special form:
`(define name function)`
 - For example:

```
(define sqr
  (lambda (x) (* x x)))
)
defines function sqr
  ■ Input: (sqr 3)
  ■ Output: 9
  ■ Input: (sqr (sqr 3))
  ■ Output: 81
```
 - ```
(define hypot
 (lambda (a b)
 (sqrt (+ (* a a) (* b b))))
)
defines function hypot
 ■ Input: (hypot 3 4)
 ■ Output: 5
```

## Bindings

- An expression can have local name-value bindings defined with the `let` special form  
(`let listofnameandvaluepairs expression`)  
where *name and value pairs* is a list of pairs (*namevalue*) and expression is returned in which each name is replaced with its value in the list
  - Input:

```
(let ((a 3)
 (b 4)
)
 (hypot a b)
)
```
  - Output: 5
- A name can be bound to a function in `let`
  - Input:

```
(let ((sqr (lambda (x) (* x x)))
 (y 3)
)
 (sqr y)
)
```
  - Output: 9

## Recursive Bindings

- An expression can have local *recursive* function bindings defined with the `letrec` special form  
(`letrec listofnameandvaluepairs expression`)  
where *name and value pairs* is a list of pairs (*namevalue*) and expression is returned where each name is replaced with its value
  - Input:

```
(letrec ((fact (lambda (n)
 (if (= n 1)
 1
 (* n (fact (- n 1)))))
)
)
 (fact 5)
)
```
  - Output: 120
  - This allows the local factorial function `fact` to refer to itself

## I/O and Sequencing

- `display` prints a value
  - Input: (`display "Hello World!"`)
  - Output: "Hello World!"
  - Input: (`display (+ 2 3)`)
  - Output: 5
- `newline` advances to a new line
  - Input: (`newline`)
- `read` returns a value from standard input
- `begin` sequences a series of expressions (its value is the value of the last expression)
  - Example:

```
(begin
 (display "Hello World!")
 (newline)
)
```
  - Example:

```
(let ((x 1)
 (y (read))
 (plus +)
)
 (begin
 (display (plus x y))
 (newline)
)
)
```

## Loops

- `do` takes a list of name-init-update triples, a termination test with final value, and a loop body  
(`do listoftriples condition body`)
- Example:

```
(do ((i 0 (+ i 1)))
 ((>= i 10) "done")
 (display i)
 (newline)
)
```
- Since everything is an expression in Scheme, a loop must return a value which in this case is the string "done"

## Higher-Order Functions

- A function is called a *higher-order function* (also called a *functional form*) if it takes a function as an argument or returns a newly constructed function as a result
- Scheme has several built-in higher-order functions, for example:
  - `apply` takes a function and a list and applies the function with the elements of the list as arguments
    - Input: `(apply '+ '(3 4))`
    - Output: 7
    - Input: `(apply (lambda (x) (* x x)) '(3))`
    - Output: 9
  - `map` takes a function and a list and returns a list after applying the function to each element of the list
    - Input: `(map odd? '(1 2 3 4))`
    - Output: `(#t () #t ())`
    - Input: `(map (lambda (x) (* x x)) '(1 2 3 4))`
    - Output: `(1 4 9 16)`
- Here is a function that applies a function to an argument twice:
  - `(define twice`  
  `(lambda (f n) (f (f n)))`  
  `)`
  - Input: `(twice sqrt 81)`
  - Output: 3

## Non-Pure Constructs: Assignments

- Assignments are considered bad in functional programming because they can change the global state of the program and possibly influence function outcomes
- `set!` assigns to a variable a new value, for example:
  - `(define a 0)`  
  `...`  
  `(set! a 1) ; overwrite a with 1`  
  `...`
  - `(let ((a 0))`  
  `(begin`  
    `...`  
    `(set! a (+ a 1)) ; increment a by 1`  
    `...`  
  `)`
- `set-car!` overwrites the head of a list
- `set-cdr!` overwrites the tail (rest) of a list

## Scheme Examples

- Recursive factorial function:

```
(define fact
 (lambda (n)
 (if (zero? n) 1 (* n (fact (- n 1)))))
)
```
- Iterative factorial function:

```
(define iterfact
 (lambda (n)
 (do ((i 1 (+ i 1))
 (f 1 (* f i)))
)
 ((> i n) f)
 ; note: loop body is omitted
)
)
)
```

## Example Recursive Functions on Lists

- Sum the elements of a list:

```
(define sum
 (lambda (lst)
 (if (null? lst)
 0
 (+ (car lst) (sum (cdr lst)))) ; add value of head to
 sum of rest of list
)
)
```

  - Input: `(sum '(1 2 3))`
  - Output: 6
- Check if element is in list:

```
(define in?
 (lambda (elt lst)
 (cond
 ((null? lst) #f) ; if list is empty, return false
 ((= elt (car lst)) #t) ; if element is the head, return
 true
 (else (in? elt (cdr lst))) ; keep searching rest of
 list
)
)
)
```

  - Input: `(in? 2 '(1 2 3))`
  - Output: `#t`

## Examples of List Functions

- `(define fill`  
  `(lambda (num elt)`  
    `(cond`  
      `((= 0 num) '())`  
      `(else (cons elt (fill (- num 1) elt)))`  
    `)`

## Examples of Higher-Order Functions

- Reduce a list by applying a binary operator to all elements (i.e.  $elt1 + elt2 + elt3 + \dots$ ):  
(define reduce  
  (lambda (op lst)  
    (if (null? (cdr lst))  
        (car lst)  
        (op (car lst) (reduce op (cdr lst))))  
  )  
)  
  o Input: (reduce + '(1 2 3))  
  o Output: 6
- Filter elements of a list for which a condition (a predicate function) returns true:  
(define filter  
  (lambda (op lst)  
    (cond  
      ((null? lst) '())  
      ((op (car lst)) (cons (car lst) (filter op (cdr lst))))  
      (else (filter op (cdr lst))))  
  )  
)  
  o Input: (filter odd? '(1 2 3 4 5))  
  o Output: (1 3 5)

- ```
)  
)  
)  
o Input: (fill 3 "a")  
o Output: ("a" "a" "a")  
• (define between  
  (lambda (start end)  
    (if (> start end)  
        '()  
        (cons start (between (+ start 1) end)))  
  )  
)  
o Input: (between 1 10)  
o Output: (1 2 3 4 5 6 7 8 9 10)  
• (define zip  
  (lambda (lst1 lst2)  
    (cond  
      ((null? lst1) '())  
      ((null? lst2) '())  
      (else (cons (list (car lst1) (car lst2)) (zip  
                          (cdr lst1) (cdr lst2))))  
    )  
  )  
)  
o Input: (zip '(1 2 3) '(a b c))  
o Output: ((1 a) (2 b) (3 c))  
• (define take  
  (lambda (num lis)  
    (cond  
      ((= num 0) '())  
      (else (cons (car lis) (take (- num 1) (cdr  
                                  lis))))  
    )  
  )  
)  
o Input: (take 3 '(a b c d e f))  
o Output: (a b c)
```

Functional Programming Today

- Significant improvements in theory and practice of functional programming have been made in recent years
 - o Strongly typed (with type inference)
 - o Modular
 - o Imperative language features that are automatically translated to functional constructs (e.g. loops by recursion)
 - o Improved efficiency
- Remaining obstacles to functional programming:
 - o *Social*: most programmers are trained in imperative programming
 - o *Commercial*: not many libraries, not very portable, and no integrated development environments for functional languages

Exercise 1: Explain the workings of a Turing Machine. Find information on the Web (for example in the Stanford Encyclopedia of Philosophy). Site your source of information.

Exercise 2: Rewrite the arithmetic expression $9 + 3 * 7 / (2 + 4) + 1$ in Scheme.

Exercise 3: Which Scheme construct(s) will cause a Scheme program to depart from a purely functional programming model?

Exercise 4: Explain the difference between a functional and a special form.

Exercise 5: Using nested lists, show how you can implement a binary tree data structure (nodes carry values and have optional left and right subtrees).

Exercise 6: Why is Scheme *homoioic*?