Name:
Course: CAP 4601
Semester: Summer 2013
Assignment: Assignment 06
Date:
08 JUL 2013
Complete the following written problems:

1. Alpha-Beta Pruning (40 Points).

Consider the following min-max tree.

a. Given that we search depth first from left to right, list all leaf nodes above that we need to search/expand. (35 Points)


Therefore, the leaf nodes that are search/expanded are 50, 25, 75, 15, 30, and 20.
b. What is the final value at the top of the tree? (5 Points)

The final value at the top of the tree is 25 .
2. The Wumpus World (70 Points).

Suppose that an agent in the Wumpus World has perceived nothing in $(1,1)$, a breeze in $(2,1)$, and a stench in (1,2):


$$
\begin{array}{ll}
\mathrm{A} & =\text { Agent } \\
\mathrm{B} & =\text { Breeze } \\
\mathrm{G} & =\text { Glitter, Gold } \\
\mathrm{OK} & =\text { Safe Square } \\
\mathrm{P} & =\text { Pit } \\
\mathrm{S} & =\text { Stench } \\
\mathrm{V} & =\text { Visited } \\
\mathrm{W} & =\text { Wumpus }
\end{array}
$$

Given this Knowledge Base, the agent now concerns itself with the contents of (1,3), (2,2), and $(3,1)$. Each of these locations can contain a pit (P). At most, one location can contain a Wumpus (W). A location can contain nothing (N).

Construct the set of all possible worlds. Each possible world should be represented by a list representing the contents of each location in the following order: $(1,3),(2,2)$, and $(3,1)$. Example: $\mathrm{N}, \mathrm{P}, \mathrm{W}$ means that there is nothing in $(1,3)$, a pit in $(2,2)$, and a Wumpus in $(3,1)$. Hint: There are 32 possible worlds. Mark the worlds in which the Knowledge Base (KB) is true and those in which each of the following sentences is true:

$$
\text { A2 = "There is not pit in }(2,2) . "
$$

$$
\mathrm{A} 3=\text { "There is a Wumpus in }(1,3) . "
$$

Hence, show that $K B \mid=A 2$ and $K B \mid=A 3$.

If the world is not supported by the KB, then mark the world False for "KB?". If the world does not support A2, then mark the world False for "A2?". If the world does not support A3, then mark the world False for "A3?".

| Number | World | KB? | A2? | A3? |
| :---: | :---: | :---: | :---: | :---: |
| 1. | N,N,N | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | True | False |
| 2. | N,N,P | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: True; $\therefore$, False | True | False |
| 3. | N,P,N | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: True; $\therefore$, False | False | False |
| 4. | N,P,P | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: True; $\therefore$, False | False | False |
| 5. | P,N,N | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | True | False |
| 6. | P,N,P | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: True; $\therefore$, False | True | False |
| 7. | P,P,N | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: True; $\therefore$, False | False | False |
| 8. | P,P,P | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: True; $\therefore$, False | False | False |
| 9. | N,N,W | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | True | False |
| 10. | N,N,P\&W | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | True | False |
| 11. | N,P,W | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | False | False |
| 12. | N,P,P\&W | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | False | False |
| 13. | P,N,W | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | True | False |
| 14. | P,N,P\&W | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | True | False |
| 15. | P,P,W | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | False | False |
| 16. | P,P,P\&W | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | False | False |
| 17. | N,W,N | In (1,2), stench and no breeze: True and, in (2,1), no stench and breeze: False; $\therefore$, False | True | False |
| 18. | N,W,P | In (1,2), stench and no breeze: True and, in (2,1), no stench and breeze: False; $\therefore$, False | True | False |
| 19. | N,P\&W,N | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | False | False |
| 20. | N,P\&W,P | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | False | False |
| 21. | P,W,N | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | True | False |
| 22. | P,W,P | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | True | False |
| 23. | P,P\&W,N | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | False | False |
| 24. | P,P\&W,P | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | False | False |
| 25. | W,N,N | In (1,2), stench and no breeze: True and, in (2,1), no stench and breeze: False; $\therefore$, False | True | True |
| 26. | W,N,P | In (1,2), stench and no breeze: True and, in (2,1), no stench and breeze: True; $\therefore$, True | True | True |
| 27. | W,P, N | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: True; $\therefore$, False | False | True |
| 28. | W,P,P | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: True; $\therefore$, False | False | True |
| 29. | P\&W,N,N | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: False; $\therefore$, False | True | True |
| 30. | P\&W,N,P | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: True; $\therefore$, False | True | True |
| 31. | P\&W,P,N | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: True; $\therefore$, False | False | True |
| 32. | P\&W,P,P | In (1,2), stench and no breeze: False and, in (2,1), no stench and breeze: True; $\therefore$, False | False | True |

Note: From Page 237, the agent will perceive a Stench in the square containing the wumpus and in directly (not diagonally) adjacent squares. Additionally, the agent will perceive a Breeze in the squares directly adjacent to a pit. If there were a pit in either $(1,3)$ or $(2,2)$, then we would perceive a Breeze in $(1,2)$; however, we do not. If there were a wumpus in either $(2,2)$ or $(3,1)$, the we would perceive a Stench in $(2,1)$; however, we do not. Therefore, we must have a wumpus only in $(1,3)$, nothing in $(2,2)$, and a pit only in $(3,1)$.
3. Propositional Logic (60 Points).

Given the following paragraph:
If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

And the following propositions:
UnicornIsMythical: The unicorn is mythical.
UnicornIsMortal: The unicorn is mortal.
UnicornIsMammal: The unicorn is a mammal.
UnicornIsHorned: The unicorn is horned.
UnicornIsMagical: The unicorn is magical.
Note: The names for the following rules are based on Figure 7.11 on Page 249 and Section 7.5
(Propositional Theorem Proving; Pages 249-254).
Below are two of the many ways these can be proved.
a. Use propositional logic to prove that the unicorn is magical. List each premise and indicate each inference rule used in your proof. You may use more or less lines than in the table below. (30 Points)

| Line | Sentence | Rule |
| :--- | :--- | :--- |
| 1. | UnicornIsMythical $\Rightarrow \neg$ UnicornIsMortal | Premise |
| 2. | $\neg$ UnicornIsMythical $\Rightarrow$ UnicornIsMortal $\wedge$ UnicornIsMammal | Premise |
| 3. | $\neg$ UnicornIsMortal $\vee$ UnicornIsMammal $\Rightarrow$ UnicornIsHorned | Premise |
| 4. | UnicornIsHorned $\Rightarrow$ UnicornIsMagical | Premise |
| 5. | $\neg$ UnicornIsMythical $\vee \neg$ UnicornIsMortal | From 1 by <br> Implication <br> Elimination |
| 6. | $\neg(\neg$ UnicornIsMythical $) \vee($ UnicornIsMortal $\wedge$ UnicornIsMammal $)$ | From 2 by <br> Implication <br> Elimination |
| 7. | UnicornIsMythical $\vee($ UnicornIsMortal $\wedge$ UnicornIsMammal $)$ | From 6 by <br> Double <br> Negation <br> Elimination |
| 8. | $\neg$ UnicornIsMortal $\vee($ UnicornIsMortal $\wedge$ UnicornIsMammal $)$ | From 5 and 7 <br> by Resolution |
| 9. | $(\neg$ UnicornIsMortal $\vee$ UnicornIsMortal $) \wedge(\neg$ UnicornIsMortal $\wedge$ UnicornIsMammal $)$ | From 8 by <br> Distributivity <br> of $\vee$ over $\wedge$ |
| 10. | $\neg$ UnicornIsMortal $\wedge$ UnicornIsMammal | From 9 by <br> Tautology |


| 11. | $\neg$ UnicornIsMortal | From 10 by <br> And <br> Elimination |
| :--- | :--- | :--- |
| 13. | $\neg(\neg$ UnicornIsMortal $\vee$ UnicornIsMammal $) \vee$ UnicornIsHorned | From 3 by <br> Implication <br> Elimination |
| 14. | $(\neg(\neg$ UnicornIsMortal $) \wedge \neg$ UnicornIsMammal $) \vee$ UnicornIsHorned | From 13 by De <br> Morgan |
| 15. | $($ UnicornIsMortal $\wedge \neg$ UnicornIsMammal $) \vee$ UnicornIsHorned | From 14 by <br> Double <br> Negation <br> Elimination |
| 16. | $($ UnicornIsMortal $\vee$ UnicornIsHorned $) \wedge(\neg$ UnicornIsMammal $\vee$ UnicornIsHorned $)$ | From 15 by <br> Distributivity <br> of $\vee$ over $\wedge$ |
| 17. | UnicornIsMortal $\vee$ UnicornIsHorned | From 16 by <br> And <br> Elimination |
| 18. | $\neg(\neg$ UnicornIsMortal $) \vee$ UnicornIsHorned | From 17 by <br> Double <br> Negation <br> Elimination |
| 19. | $\neg$ UnicornIsMortal $\Rightarrow$ UnicornIsHorned | From 18 by <br> Implication <br> Elimination |
| 20. | UnicornIsHorned | From 11 and <br> 19 by Modus <br> Ponens |
|  |  | From 4 and 20 <br> by Modus <br> Ponens |

b. Use propositional logic to prove that the unicorn is horned. List each premise and indicate each inference rule used in your proof. You may use more or less lines than in the table below. (30 Points)

| Line | Sentence | Rule |
| :--- | :--- | :--- |
| 1. | UnicornIsMythical $\Rightarrow \neg$ UnicornIsMortal | Premise |
| 2. | $\neg$ UnicornIsMythical $\Rightarrow$ UnicornIsMortal $\wedge$ UnicornIsMammal | Premise |
| 3. | $\neg$ UnicornIsMortal $\vee$ UnicornIsMammal $\Rightarrow$ UnicornIsHorned | Premise |
| 4. | UnicornIsHorned $\Rightarrow$ UnicornIsMagical | Premise |
| 5. | $\neg$ UnicornIsMythical $\vee \neg$ UnicornIsMortal | From 1 by <br> Implication <br> Elimination |


| 6. | $\neg(\neg$ UnicornIsMythical $) \vee($ UnicornIsMortal $\wedge$ UnicornIsMammal $)$ | From 2 by Implication Elimination |
| :---: | :---: | :---: |
| 7. | UnicornIsMythical $\vee($ UnicornIsMortal $\wedge$ UnicornIsMammal $)$ | From 6 by <br> Double <br> Negation <br> Elimination |
| 8. | $\neg$ UnicornIsMortal $\vee($ UnicornIsMortal $\wedge$ UnicornIsMammal $)$ | From 5 and 7 by Resolution |
| 9. | $(\neg$ UnicornIsMortal $\vee$ UnicornIsMortal $) \wedge(\neg$ UnicornIsMortal $\wedge$ UnicornIsMammal $)$ | From 8 by Distributivity of $\vee$ over $\wedge$ |
| 10. | $\neg$ UnicornIsMortal $\wedge$ UnicornIsMammal | From 9 by <br> Tautology |
| 11. | $\neg$ UnicornIsMortal | From 10 by And <br> Elimination |
| 13. | $\neg(\neg$ UnicornIsMortal $\vee$ UnicornIsMammal $) \vee$ UnicornIsHorned | From 3 by Implication Elimination |
| 14. | $(\neg(\neg$ UnicornIsMortal $) \wedge \neg$ UnicornIsMammal $) \vee$ UnicornIsHorned | From 13 by De Morgan |
| 15. | (UnicornIsMortal $\wedge \neg$ UnicornIsMammal $) \vee$ UnicornIsHorned | From 14 by <br> Double <br> Negation <br> Elimination |
| 16. | $($ UnicornIsMortal $\vee$ UnicornIsHorned $) \wedge(\neg$ UnicornIsMammal $\vee$ UnicornIsHorned $)$ | From 15 by Distributivity of $\vee$ over |
| 17. | UnicornIsMortal $\vee$ UnicornIsHorned | From 16 by And <br> Elimination |
| 18. | $\neg(\neg$ UnicornIsMortal $) \vee$ UnicornIsHorned | From 17 by <br> Double <br> Negation <br> Elimination |
| 19. | $\neg$ UnicornIsMortal $\Rightarrow$ UnicornIsHorned | From 18 by Implication Elimination |
| 20. | UnicornIsHorned | From 11 and 19 by Modus Ponens |

## 4. Conjunctive Normal Form (50 Points).

Consider the following sentence:

$$
[(\text { Food } \Rightarrow \text { Party }) \vee(\text { Drinks } \Rightarrow \text { Party })] \Rightarrow[(\text { Food } \wedge \text { Drinks }) \Rightarrow \text { Party }]
$$

a. Using the procedure starting on page 253 , convert this sentence into Conjunctive Normal Form showing each step. (Points 40)
(1) Eliminate $\Leftrightarrow$, replacing with $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge(\beta \Rightarrow \alpha)$ :

$$
[(\text { Food } \Rightarrow \text { Party }) \vee(\text { Drinks } \Rightarrow \text { Party })] \Rightarrow[(\text { Food } \wedge \text { Drinks }) \Rightarrow \text { Party }]
$$

(2) Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg \alpha \vee \beta$ :

$$
\begin{gathered}
{[(\text { Food } \Rightarrow \text { Party }) \vee(\text { Drinks } \Rightarrow \text { Party })] \Rightarrow[(\text { Food } \wedge \text { Drinks }) \Rightarrow \text { Party }]} \\
{[(\neg \text { Food } \vee \text { Party }) \vee(\text { Drinks } \Rightarrow \text { Party })] \Rightarrow[(\text { Food } \wedge \text { Drinks }) \Rightarrow \text { Party }]} \\
{[(\neg \text { Food } \vee \text { Party }) \vee(\neg \text { Drinks } \vee \text { Party })] \Rightarrow[(\text { Food } \wedge \text { Drinks }) \Rightarrow \text { Party }]} \\
{[(\neg \text { Food } \vee \text { Party }) \vee(\neg \text { Drinks } \vee \text { Party })] \Rightarrow[\neg(\text { Food } \wedge \text { Drinks }) \vee \text { Party }]} \\
\\
\neg[(\neg \text { Food } \vee \text { Party }) \vee(\neg \text { Drinks } \vee \text { Party })] \vee[\neg(\text { Food } \wedge \text { Drinks }) \vee \text { Party }]
\end{gathered}
$$

(3) CNF requires $\neg$ to appear only in literals, so we "move $\neg$ inwards" by repeated application of the following equivalences: $\neg(\neg \alpha) \equiv \alpha, \neg(\alpha \wedge \beta) \equiv \neg \alpha \vee \neg \beta$, and $\neg(\alpha \vee \beta) \equiv \neg \alpha \wedge \neg \beta$.

$$
\begin{aligned}
& \neg[(\neg \text { Food } \vee \text { Party }) \vee(\neg \text { Drinks } \vee \text { Party })] \vee[\neg(\text { Food } \wedge \text { Drinks }) \vee \text { Party }] \\
& \neg[(\neg \text { Food } \vee \text { Party }) \vee(\neg \text { Drinks } \vee \text { Party })] \vee[[(\neg \text { Food } \vee \neg \text { Drinks }) ~\vee \text { Party }] \\
& {[\neg(\neg \text { Food } \vee \text { Party }) \wedge \neg(\neg \text { Drinks } \vee \text { Party })] } \\
& {[(\neg \neg \text { Food } \wedge \neg \text { Party })} \\
&(\neg \neg(\neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party })] \vee[\neg \text { Food } \vee \neg \neg \text { Drinks } \vee \text { Party }] \\
& {[(\text { Food } \wedge \neg \text { Party })] \wedge \neg(\neg \text { Drinks } \vee \text { Party })] \vee[\neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party }] } \\
& {[(\text { Food } \wedge \neg \text { Party }) \wedge(\neg \neg \text { Drinks } \wedge \neg \text { Party })] \vee[\neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party }] } \\
& {[(\text { Food } \wedge \neg \text { Party }) \wedge(\text { Drinks } \wedge \neg \text { Party })] \vee[\neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party }] }
\end{aligned}
$$

(4) Now we have a sentence containing nested $\wedge$ and $\vee$ operators applied to literals. We apply the distributivity law (distributing $\vee$ over $\wedge$ wherever possible):

$$
\left.\begin{array}{c}
{[(\text { Food } \wedge \neg \text { Party }) \wedge(\text { Drinks } \wedge \neg \text { Party })] \vee[\neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party }]} \\
(\text { Food } \wedge \neg \text { Party } \wedge \text { Drinks } \wedge \neg \text { Party }) \vee(\neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party }) \\
\\
\left(\begin{array}{c}
(\text { Food } \vee(\neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party })) \\
\wedge \\
(\neg \text { Party } \vee(\neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party })) \\
\wedge \\
(\text { Drinks } \vee(\neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party })) \\
\wedge \\
(\neg \text { Party } \vee(\neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party }))
\end{array}\right) \\
\\
\left(\begin{array}{c}
(\text { Food } \vee \neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party }) \\
\wedge \\
(\neg \text { Party } \vee \neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party }) \\
\wedge \\
(\text { Drinks } \vee \neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party }) \\
\wedge \\
(\neg \text { Party } \vee \neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party })
\end{array}\right) \\
\left(\begin{array}{c}
(\text { Food } \vee \neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party }) \\
\wedge \\
(\neg \text { Party } \vee \text { Party } \vee \neg \text { Food } \vee \neg \text { Drinks }) \\
\wedge \\
(\text { Drinks } \vee \neg \text { Drinks } \vee \neg \text { Food } \vee \text { Party }) \\
\wedge \\
(\neg \text { Party } \vee \text { Party } \vee \neg \text { Food } \vee \neg \text { Drinks })
\end{array}\right)
\end{array}\right)
$$

b. Using resolution, determine if this sentence is valid, satisfiable (but not valid), or unsatisfiable. (Points 10)

$$
\begin{aligned}
& \left(\begin{array}{c}
(\text { Food } \vee \neg \text { Food } \vee \neg \text { Drinks } \vee \text { Party }) \\
\wedge \\
(\neg \text { Party } \vee \text { Party } \vee \neg \text { Food } \vee \neg \text { Drinks }) \\
\wedge \\
(\text { Drinks } \vee \neg \text { Drinks } \vee \neg \text { Food } \vee \text { Party }) \\
\wedge \\
(\neg \text { Party } \vee \text { Party } \vee \neg \text { Food } \vee \neg \text { Drinks })
\end{array}\right)
\end{aligned}
$$

$$
\begin{aligned}
& ((\text { True } \vee \neg \text { Drinks } \vee \text { Party })) \\
& \wedge \\
& (\text { True } \vee \neg \text { Food } \vee \neg \text { Drinks }) \\
& \wedge \\
& (\text { True } \vee \neg \text { Food } \vee \text { Party }) \\
& \wedge \\
& ((\text { True } \vee \neg \text { Food } \vee \neg \text { Drinks })) \\
& (T \text { True } \wedge \text { True } \wedge \text { True } \wedge \widehat{\text { True }}) \\
& \text { True }
\end{aligned}
$$

Therefore, this sentence is valid.
5. Resolution (40 Points).

A propositional 2-CNF expression is a conjunction of clauses, each containing exactly 2 literals, e.g.,

$$
(A \vee B) \wedge(\neg A \vee C) \wedge(\neg B \vee D) \wedge(\neg C \vee G) \wedge(\neg D \vee G)
$$

Prove using resolution that the above sentence entails $G$.

| Line | Sentence | Rule |
| :--- | :--- | :--- |
| 1. | $(A \vee B) \wedge(\neg A \vee C) \wedge(\neg B \vee D) \wedge(\neg C \vee G) \wedge(\neg D \vee G)$ |  |
| 2. | $A \vee B$ | From 1 by And Elimination |
| 3. | $\neg A \vee C$ | From 1 by And Elimination |
| 4. | $B \vee C$ | From 2 and 3 by Resolution |
| 5. | $\neg B \vee D$ | From 1 by And Elimination |
| 6. | $C \vee D$ | From 4 and 5 by Resolution |
| 7. | $\neg C \vee G$ | From 1 by And Elimination |
| 8. | $D \vee G$ | From 6 and 7 by Resolution |
| 9. | $\neg D \vee G$ | From 1 by And Elimination |
| 10. | $G \vee G$ | From 8 and 9 by Resolution |
| 11. | $G$ | From 10 by Logical <br> Equivalence |

6. First Order Logic (100 Points).

Given the following vocabulary with the following symbols:
Student $(x)$ : Predicate. Person $x$ is a student.
$\operatorname{Knows}(x, y)$ : Predicate. Student $x$ knows concept $y$.
Course $(x)$ : Predicate. Subject $x$ is a course.

Covers $(x, y)$ : Predicate. Course $x$ covers concept $y$.
Amy, Brian: Constants denoting people.
MAC1140 : Constants denoting the course College Algebra.
MatrixMethods : Constant denoting the concept of matrix methods.
Convert the following sentences to first-order logic:
a. Amy is a student and knows matrix methods. (5 Points)

$$
\text { Student }(\text { Amy }) \wedge \text { Knows }(\text { Amy, MatrixMethods })
$$

b. Some student knows matrix methods. (10 Points)

$$
\exists x, \text { Student }(x) \wedge \text { Knows }(x, \text { MatrixMethods })
$$

c. Every student takes MAC 1140. (10 Points)

$$
\forall x, \text { Student }(x) \Rightarrow \text { Takes }(x, M A C 1140)
$$

d. MAC 1140 is a course that the student, Brian, has not taken. (10 Points)

$$
\text { Student }(\text { Brian }) \wedge \neg \text { Taken }(\text { Brian }, M A C 1140)
$$

e. There is some course that every student has not taken. (20 Points)

$$
\exists x, \operatorname{Course}(x) \wedge[\forall y, \operatorname{Student}(y) \Rightarrow \neg \text { Taken }(y, x)]
$$

f. If Brian is a student, takes the course MAC 1140, and MAC 1140 covers matrix methods, then Brian knows matrix methods. (15 Points)

Student $($ Brian $) \wedge$ Takes $($ Brian, MAC1140 $) \wedge$ Covers $($ MAC1140,MatrixMethods $) \Rightarrow$ Knows $($ Brian, MatrixMethods $)$
g. If a student takes a course and the course covers some concept, then the student knows that concept. (30 Points)

$$
\forall x, \forall y, \forall z, \operatorname{Student}(x) \wedge \operatorname{Course}(y) \wedge \operatorname{Takes}(x, y) \wedge \operatorname{Covers}(y, z) \Rightarrow \operatorname{Knows}(x, z)
$$

7. First Order Logic (90 Points).

This exercise uses the function MapColor and predicates $\operatorname{In}(x, y), \operatorname{Borders}(x, y)$, and Country $(x)$, whose arguments are geographical regions, along with constant symbols for various regions. In each of the following, we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence, (2) is syntactically invalid and therefore meaningless, or (3) is syntactically valid but does not express the meaning of the English sentence.
a. Paris and Marseilles are both in France.
(i) In (Paris $\wedge$ Marseilles, France) (10 Points)
(2) syntactically invalid and therefore meaningless
(ii) In $($ Paris, France $) \wedge$ In $($ Marseilles, France) (10 Points)
(1) correctly expresses the English sentence
(iii) In(Paris, France) $\vee \operatorname{In}($ Marseilles, France) (10 Points)
(3) is syntactically valid but does not express the meaning of the English sentence

NOTE: This incorrectly reads: Either Paris is in France, Marseilles is in France, or both are in France.
b. There is a country that borders both Iraq and Pakistan.
(i) $\exists c$ Country $(c) \wedge \operatorname{Border}(c$, Iraq $) \wedge \operatorname{Border}(c$, Pakistan $)(10$ Points)
(1) correctly expresses the English sentence
(ii) $\quad \exists c \quad$ Country $(c) \Rightarrow[\operatorname{Border}(c, \operatorname{Iraq}) \wedge \operatorname{Border}(c$, Pakistan $)]$ (10 Points)
(3) is syntactically valid but does not express the meaning of the English sentence

NOTE: This incorrectly reads: If there is a country, then that country borders Iraq and Pakistan.
(iii) $\quad[\exists c \quad$ Country $(c)] \Rightarrow[\operatorname{Border}(c$, Iraq $) \wedge$ Border $(c$, Pakistan $)]$ (10 Points) (2) syntactically invalid and therefore meaningless
c. All countries that border Ecuador are in South America.
(i) $\quad \forall c$ Country $(c) \wedge$ Border $(c$, Ecuador $) \wedge$ In $(c$, SouthAmerica) (10 Points)
3) is syntactically valid but does not express the meaning of the English sentence

NOTE: This reads: Every country borders Ecuador and is in South America.
(ii) $\quad \forall c \quad$ Country $(c) \Rightarrow[\operatorname{Border}(c$, Ecuador $) \Rightarrow \operatorname{In}(c$, SouthAmerica $)]$ (10 points)
(1) correctly expresses the English sentence

NOTE: This translates to:

$$
\begin{array}{cc}
\forall c & \text { Country }(c) \Rightarrow[\text { Border }(c, \text { Ecuador }) \Rightarrow \operatorname{In}(c, \text { SouthAmerica })] \\
\forall c & \neg \text { Country }(c) \vee[\neg \text { Border }(c, \text { Ecuador }) \vee \operatorname{In}(c, \text { SouthAmerica })] \\
\forall c & {[\neg \text { Country }(c) \vee \neg \text { Border }(c, \text { Ecuador })] \vee \operatorname{In}(c, \text { SouthAmerica })} \\
\forall c & \neg[\text { Country }(c) \wedge \text { Border }(c, \text { Ecuador })] \vee \operatorname{In}(c, \text { SouthAmerica }) \\
\forall c & \text { Country }(c) \wedge \text { Border }(c, \text { Ecuador }) \Rightarrow \operatorname{In}(c, \text { SouthAmerica })
\end{array}
$$

(iii) $\quad \forall c \quad$ Country $(c) \wedge \operatorname{Border}(c$, Ecuador $) \Rightarrow \operatorname{In}(c$, SouthAmerica) (10 Points)
(1) correctly expresses the English sentence
8. Research Project (50 Points).
a. Write a rough draft of the title of your research project. (10 Points)
b. Write a rough draft of the abstract of your research project. (40 Points)

This assignment has no programming problems.
After completing Assignment 06, create an assignment_06_lastname.pdf file for your written assignment.

Upload your assignment_06_lastname.pdf file for your written assignment to the Assignment 06 location on the BlackBoard site: https:/ / campus.fsu.edu.

