Assignment 1: Answers to Written Problems.

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1. Terminology (50 Points):

Given the following task environments, enter their properties/characteristics. The properties/characteristics of the Checkers task environment is provided as an example.

Task Environment	**Fully Observable or Partially Observable**	**Single Agent or Multiagent**	**Deterministic or Stochastic**	**Episodic or Sequential**	**Static or Dynamic**	**Discrete or Continuous**	**Know or Unknown**
Checkers	Fully Observable	Multiagent	Deterministic	Sequential	Static	Discrete	Known
Chess (without a clock)	Fully Observable	Multiagent	Deterministic	Sequential	Static	Discrete	Known
Poker	Partially Observable	Multiagent	Stocastic	Sequential	Static	Discrete	Known
Human Driving	Partially Observable	Multiagent	Stochastic	Sequential	Dynamic	Continuous	Known
Robotic Car (Machine Driving)	Partially Observable	Multiagent	Stochastic	Sequential	Dynamic	Continuous	Known
Medical Diagnosis	Partially Observable	Single Agent	Stochastic	Sequential	Dynamic	Continuous	Unknown
Image Analysis	Fully Observable	Single Agent	Deterministic	Episodic	Static	Continuous	Unknown
Newborn Baby	Partially Observable	Multiagent	Stochastic	Sequential	Dynamic	Continuous	Unknown

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2. Mean and Standard Deviation (20 Points): Given the following test scores, calculate the Sample Mean, the Biased Sample Standard Deviation, and the Unbiased Sample Standard Deviation.

Scores: `{ 60, 70, 80, 90, 100, 70, 80, 90, 75, 85, 80 }`

Show all work and give exact answers (i.e. roots, fractions, etc.), not decimal approximations.

For the 11 grades above, the mean is:

$$\begin{aligned}\overline{x} &= \frac{\sum_{i=1}^{n} x_i}{n} \\ &= \frac{60+70+80+90+100+70+80+90+75+85+80}{11} \\ &= \frac{880}{11} \\ &= 80\end{aligned}$$

The sum of squared differences is:

$$
\begin{aligned}
\mathrm{SSD} &= \sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 \\
&= \begin{pmatrix} (60-80)^2 + (70-80)^2 + (80-80)^2 + (90-80)^2 + (100-80)^2 + (70-80)^2 + \\ (80-80)^2 + (90-80)^2 + (75-80)^2 + (85-80)^2 + (80-80)^2 \end{pmatrix} \\
&= \begin{pmatrix} (-20)^2 + (-10)^2 + (0)^2 + (10)^2 + (20)^2 + (-10)^2 + \\ (0)^2 + (10)^2 + (-5)^2 + (5)^2 + (0)^2 \end{pmatrix} \\
&= \begin{pmatrix} 400 + 100 + 0 + 100 + 400 + 100 + \\ 0 + 100 + 25 + 25 + 0 \end{pmatrix} \\
&= 1250
\end{aligned}
$$

The biased sample variance is:

$$
\begin{aligned}
s_{\text{biased}}^2 &= \frac{\mathrm{SSD}}{n} \\
&= \frac{1250}{11}
\end{aligned}
$$

Aside: this biased sample variance is approximately 113.636.

Hence, the biased sample standard deviation is:

$$
\begin{aligned}
s_{\text{biased}} &= \sqrt{s_{\text{biased}}^2} \\
&= \sqrt{\frac{1250}{11}} \\
&= \sqrt{\frac{2 \cdot 625}{11}} \\
&= \sqrt{\frac{2 \cdot 25 \cdot 25}{11}} \\
&= 25\sqrt{\frac{2}{11}}
\end{aligned}
$$

Aside: this biased sample standard deviation is approximately 10.66.

The unbiased sample variance is:

$$
\begin{aligned}
s_{\text{unbiased}}^2 &= \frac{\mathrm{SSD}}{n-1} \\
&= \frac{1250}{11-1} \\
&= \frac{1250}{10} \\
&= 125
\end{aligned}
$$

Hence, the unbiased sample standard deviation is:

$$\begin{aligned}s_{\text{unbiased}} &= \sqrt{s_{\text{unbiased}}^2}\\ &= \sqrt{125}\\ &= \sqrt{5\cdot 25}\\ &= \sqrt{5\cdot 5\cdot 5}\\ &= 5\sqrt{5}\end{aligned}$$

Aside: this unbiased sample standard deviation is approximately 11.1803.

Therefore, we have:

Name	**Value**
Sample Mean:	$\bar{x} = 80$
Biased Sample Standard Deviation:	$s_{\text{biased}} = 25\sqrt{\frac{2}{11}}$ or I will accept $s_{\text{biased}} = \sqrt{\frac{1250}{11}}$
Unbiased Sample Standard Deviation:	$s_{\text{unbiased}} = 5\sqrt{5}$ or I will accept $s_{\text{unbiased}} = \sqrt{125}$

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3. Regression (30 Points): Given the following partial radar output where the first parameter is bearing in degrees and the second parameter is range in nautical miles, convert from Polar coordinates to Cartesian coordinates and perform a linear regression on this data.

Radar Data: $\left\{(0°,5\text{nm}),\left(45°,\frac{5}{\sqrt{2}}\text{nm}\right),(90°,5\text{nm})\right\}$

Show all work and give exact answers (i.e. roots, fractions, etc.), not decimal approximations.

First, we convert to Cartesian coordinates:

$$x = r\cdot\cos(\theta)$$
$$y = r\cdot\sin(\theta)$$

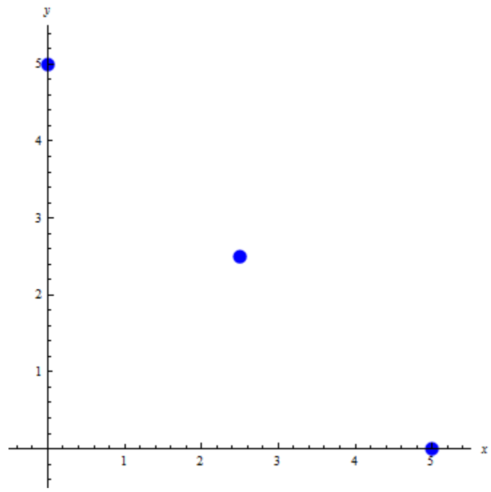
where r is the range and θ is the bearing. Hence, we have:

$$\begin{aligned}(0°,5\text{nm}) &\Rightarrow (5\text{nm}\cdot\cos(0°),5\text{nm}\cdot\sin(0°))\\ &\Rightarrow (5\text{nm}\cdot 1,5\text{nm}\cdot 0)\\ &\Rightarrow (5\text{nm},0\text{nm})\end{aligned}$$

$$\begin{aligned}\left(45°,\frac{5}{\sqrt{2}}\text{nm}\right) &\Rightarrow \left(\frac{5}{\sqrt{2}}\text{nm}\cdot\cos(45°),\frac{5}{\sqrt{2}}\text{nm}\cdot\sin(45°)\right)\\ &\Rightarrow \left(\frac{5}{\sqrt{2}}\text{nm}\cdot\frac{\sqrt{2}}{2},\frac{5}{\sqrt{2}}\text{nm}\cdot\frac{\sqrt{2}}{2}\right)\\ &\Rightarrow \left(\frac{5}{2}\text{nm},\frac{5}{2}\text{nm}\right)\end{aligned}$$

$$
\begin{aligned}
(90°, 5\text{nm}) &\Rightarrow (5\text{nm}\cdot\cos(90°), 5\text{nm}\cdot\sin(90°)) \\
&\Rightarrow (5\text{nm}\cdot 0, 5\text{nm}\cdot 1) \\
&\Rightarrow (0\text{nm}, 5\text{nm})
\end{aligned}
$$

Hence, we have these points:



For these 3 points, the mean is:

$$
\begin{aligned}
\text{mean} &= \left(\frac{\sum_{i=1}^{n} x_i}{n}, \frac{\sum_{i=1}^{n} y_i}{n}\right) \\
&= \left(\frac{5\text{nm} + \frac{5}{2}\text{nm} + 0\text{nm}}{3}, \frac{0\text{nm} + \frac{5}{2}\text{nm} + 5\text{nm}}{3}\right) \\
&= \left(\frac{\frac{15}{2}\text{nm}}{3}, \frac{\frac{15}{2}\text{nm}}{3}\right) \\
&= \left(\frac{5\cdot 3}{2\cdot 3}\text{nm}, \frac{5\cdot 3}{2\cdot 3}\text{nm}\right) \\
&= \left(\frac{5}{2}\text{nm}, \frac{5}{2}\text{nm}\right)
\end{aligned}
$$

Hence, $\bar{x} = \frac{5}{2}\text{nm}$ and $\bar{y} = \frac{5}{2}\text{nm}$.

The sum of squared differences for the x's is:

$$\begin{aligned}\mathrm{SSD}_{xx} &= \sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 \\ &= \left(5\mathrm{nm} - \frac{5}{2}\mathrm{nm}\right)^2 + \left(\frac{5}{2}\mathrm{nm} - \frac{5}{2}\mathrm{nm}\right)^2 + \left(0\mathrm{nm} - \frac{5}{2}\mathrm{nm}\right)^2 \\ &= \left(\frac{5}{2}\mathrm{nm}\right)^2 + \left(0\mathrm{nm}\right)^2 + \left(-\frac{5}{2}\mathrm{nm}\right)^2 \\ &= \frac{25}{4}\mathrm{nm}^2 + 0\mathrm{nm}^2 + \frac{25}{4}\mathrm{nm}^2 \\ &= \frac{50}{4}\mathrm{nm}^2\end{aligned}$$

The sum of product of differences for the x's and the y's is:

$$\begin{aligned}\mathrm{SPD}_{xy} &= \sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right) \\ &= \left(5\mathrm{nm} - \frac{5}{2}\mathrm{nm}\right)\left(0\mathrm{nm} - \frac{5}{2}\mathrm{nm}\right) + \left(\frac{5}{2}\mathrm{nm} - \frac{5}{2}\mathrm{nm}\right)\left(\frac{5}{2}\mathrm{nm} - \frac{5}{2}\mathrm{nm}\right) + \left(0\mathrm{nm} - \frac{5}{2}\mathrm{nm}\right)\left(5\mathrm{nm} - \frac{5}{2}\mathrm{nm}\right) \\ &= \left(\frac{5}{2}\mathrm{nm}\right)\left(-\frac{5}{2}\mathrm{nm}\right) + \left(0\mathrm{nm}\right)\left(0\mathrm{nm}\right) + \left(-\frac{5}{2}\mathrm{nm}\right)\left(\frac{5}{2}\mathrm{nm}\right) \\ &= \left(-\frac{25}{4}\mathrm{nm}^2\right) + \left(0\mathrm{nm}^2\right) + \left(-\frac{25}{4}\mathrm{nm}^2\right) \\ &= -\frac{50}{4}\mathrm{nm}^2\end{aligned}$$

Hence, the weight is:

$$\begin{aligned}w &= \frac{\mathrm{SPD}_{xy}}{\mathrm{SSD}_{xx}} \\ &= \frac{-\frac{50}{4}\mathrm{nm}^2}{\frac{50}{4}\mathrm{nm}^2} \\ &= -1\end{aligned}$$

Since the weight is a synonym for the slope, then the slope is:

$$\begin{aligned}m &= w \\ &= -1\end{aligned}$$

Since the intercept is $b = -w\overline{x} + \overline{y}$, then we have:

$$\begin{aligned}b &= -w\overline{x} + \overline{y} \\ &= -\left(-1\right)\left(\frac{5}{2}\mathrm{nm}\right) + \frac{5}{2}\mathrm{nm} \\ &= \frac{10}{2}\mathrm{nm} \\ &= 5\mathrm{nm}\end{aligned}$$

Therefore, the linear regression is:

$$\begin{aligned} y &= mx + b \\ &= (-1)x + (5\text{nm}) \\ &= -x + 5\text{nm} \end{aligned}$$

and we have:

Name	**Value**
Mean:	$\text{mean} = \left(\frac{5}{2}\text{nm}, \frac{5}{2}\text{nm}\right)$
$m = w$ (Slope):	$m = -1$
$b = -w\bar{x} + \bar{y}$ (Intercept):	$b = 5\text{nm}$

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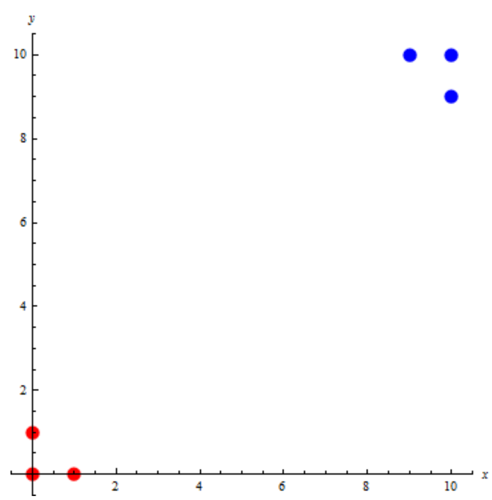
4. Level Set/Normal Vector (50 Points): Given the following two classes each with three points, find the mean of each class and then find the equation of the plane that separates those classes halfway between their respective means. In other words, find the separating plane that could be used for classification with the maximum margin between the classes.

Class 1 Points: `{ { 0, 1 }, { 1, 0 }, { 0, 0 } }`

Class 2 Points: `{ { 10, 10 }, { 10, 9 }, { 9, 10 } }`

Show all work and give exact answers (i.e. roots, fractions, etc.), not decimal approximations.

Let Class 1 be red points and Class 2 be blue points, then we have:



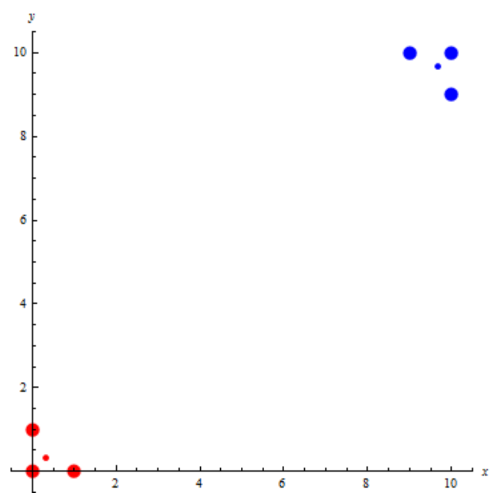
For the 3 Class 1 Points, the mean is:

$$\begin{aligned}
\text{mean}_{\text{Class 1}} &= \left(\frac{\sum_{i=1}^{n} x_i}{n}, \frac{\sum_{i=1}^{n} y_i}{n} \right) \\
&= \left(\frac{0+1+0}{3}, \frac{1+0+0}{3} \right) \\
&= \left(\frac{1}{3}, \frac{1}{3} \right)
\end{aligned}$$

For the 3 Class 2 Points, the mean is:

$$\begin{aligned}
\text{mean}_{\text{Class 2}} &= \left(\frac{\sum_{i=1}^{n} x_i}{n}, \frac{\sum_{i=1}^{n} y_i}{n} \right) \\
&= \left(\frac{10+10+9}{3}, \frac{10+9+10}{3} \right) \\
&= \left(\frac{29}{3}, \frac{29}{3} \right)
\end{aligned}$$

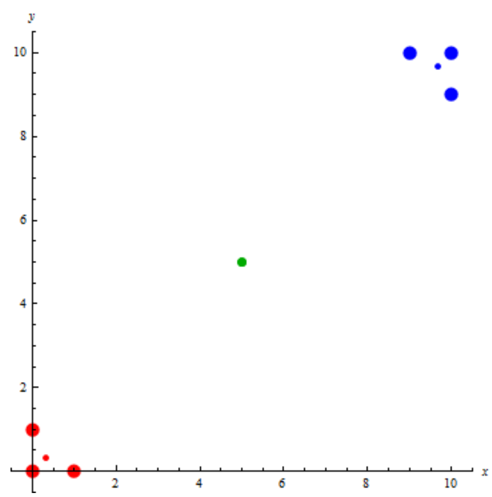
Hence, we have:



The midpoint between the mean of Class 1 and the mean of Class 2 is:

$$\begin{aligned}
\text{midpoint} &= \frac{\left(\frac{1}{3},\frac{1}{3}\right)+\left(\frac{29}{3},\frac{29}{3}\right)}{2} \\
&= \frac{\left(\frac{1}{3}+\frac{29}{3},\frac{1}{3}+\frac{29}{3}\right)}{2} \\
&= \frac{\left(\frac{30}{3},\frac{30}{3}\right)}{2} \\
&= \frac{(10,10)}{2} \\
&= \left(\frac{10}{2},\frac{10}{2}\right) \\
&= (5,5)
\end{aligned}$$

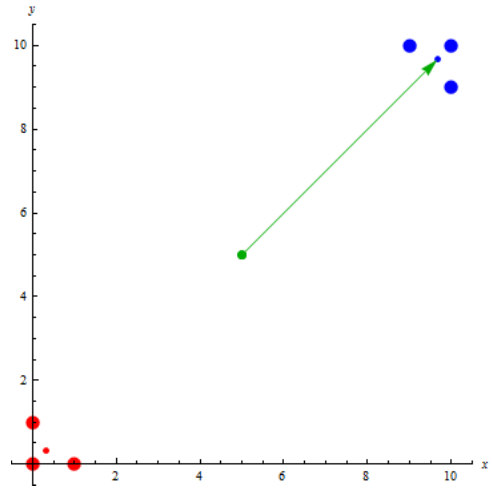
Hence, we have:



Next, we find the vector that points in the direction from the midpoint to the blue Class 2 mean:

$$\begin{aligned}
\vec{\mathbf{n}} &= \text{mean}_{\text{Class 2}} - \text{midpoint} \\
&= \left(\frac{29}{3},\frac{29}{3}\right)-(5,5) \\
&= \left(\frac{14}{3},\frac{14}{3}\right)
\end{aligned}$$

Hence, we have:



Next, we normalize that vector:

$$\vec{\mathbf{n}}_{\text{normalized}} = \frac{\vec{\mathbf{n}}}{\left\|\vec{\mathbf{n}}\right\|}$$

$$= \frac{\left(\frac{14}{3}, \frac{14}{3}\right)}{\sqrt{\left(\frac{14}{3}\right)^2 + \left(\frac{14}{3}\right)^2}}$$

$$= \frac{\left(\frac{14}{3}, \frac{14}{3}\right)}{\sqrt{\frac{196}{9} + \frac{196}{9}}}$$

$$= \frac{\left(\frac{14}{3}, \frac{14}{3}\right)}{\sqrt{\frac{2 \cdot 2 \cdot 2 \cdot 7 \cdot 7}{3 \cdot 3}}}$$

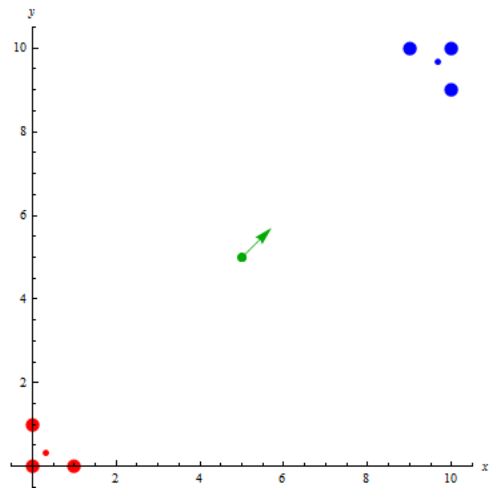
$$= \frac{\left(\frac{14}{3}, \frac{14}{3}\right)}{\frac{14}{3}\sqrt{2}}$$

$$= \left(\frac{\frac{14}{3}}{\frac{14}{3}\sqrt{2}}, \frac{\frac{14}{3}}{\frac{14}{3}\sqrt{2}}\right)$$

$$= \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$= \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

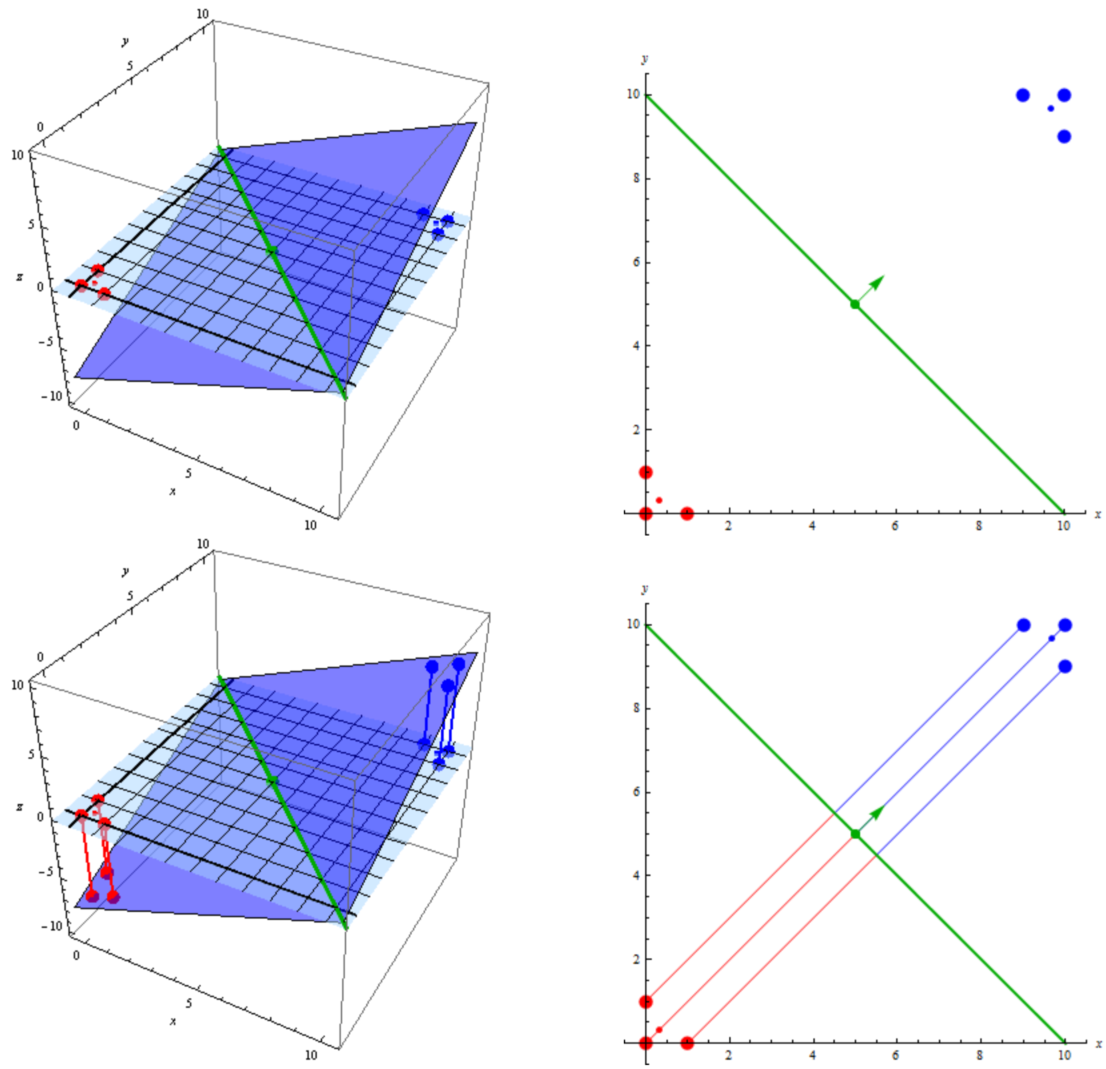
Hence, we have:



Next, we find the equation of the separating plane:

$$
\begin{aligned}
f(x,y) &= \vec{\mathbf{n}}\bullet(\mathbf{x}-\mathbf{x}_0) \\
&= \left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)\bullet\big((x,y)-(5,5)\big) \\
&= \left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)\bullet(x-5,y-5) \\
&= \frac{\sqrt{2}}{2}(x-5)+\frac{\sqrt{2}}{2}(y-5) \\
&= \frac{\sqrt{2}}{2}x-\frac{5\sqrt{2}}{2}+\frac{\sqrt{2}}{2}y-\frac{5\sqrt{2}}{2} \\
&= \frac{\sqrt{2}}{2}x+\frac{\sqrt{2}}{2}y-5\sqrt{2}
\end{aligned}
$$

Hence, we have:



Note: If the normalized normal vector were pointing in the opposite direction, then that would also be acceptable. In that case, both the Level Set graph (i.e. the graph of the plane) and the graph of the Normal Vector would just be opposite of what they are above.

Name	**Value**
Mean of Class 1	$\text{mean}_{\text{Class 1}} = \left(\frac{1}{3}, \frac{1}{3}\right)$
Mean of Class 2	$\text{mean}_{\text{Class 2}} = \left(\frac{29}{3}, \frac{29}{3}\right)$
Midpoint	$\text{midpoint} = (5,5)$

Normalized Normal Vector	$\vec{\mathbf{n}}_{\text{normalized}} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$ or $\vec{\mathbf{n}}_{\text{normalized}} = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right)$
Equation of the Separating Plane	$f(x,y) = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 5\sqrt{2}$ or $f(x,y) = -\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y + 5\sqrt{2}$

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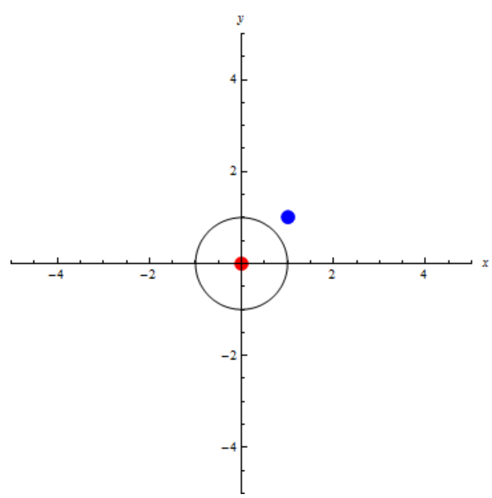
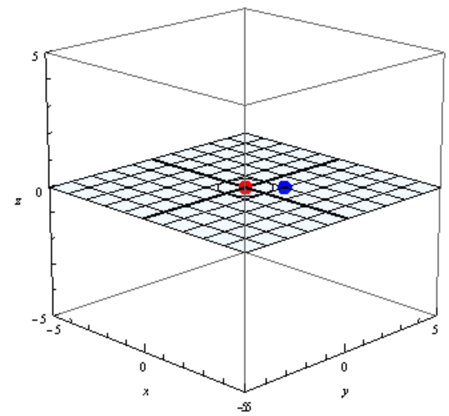
5. Level Set (20 Points): Given the equation $x^2 + y^2 = 1$ (a circle with a radius of one) and using a level set, design a single if statement that classifies points into two classes: (1) those points on or in the circle and (2) those points outside the circle. Show that `{ 0, 0 }` is in that circle, but that `{ 1, 1 }` is not.
Show all work and give exact answers (i.e. roots, fractions, etc.), not decimal approximations.

Here is what we want:



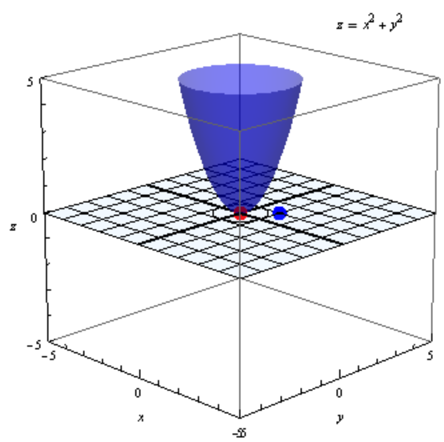
In order to get that, we look to the next highest dimension:

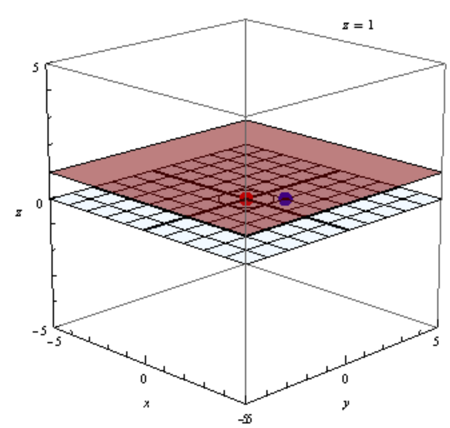


and we use a level set. Specifically, we let z equal both sides of the equation:

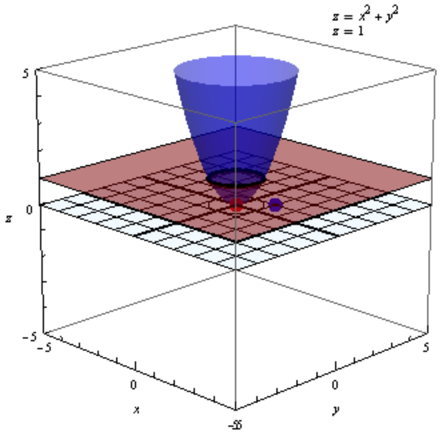
$$\underbrace{x^2 + y^2}_{\text{Let } z = x^2 + y^2} = \underset{\text{Let } z=1}{1}$$

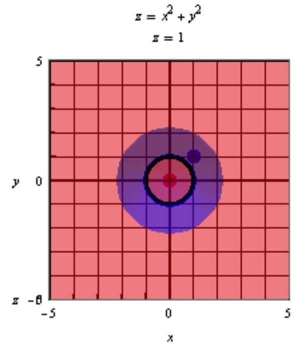
such that we have the following graphs:

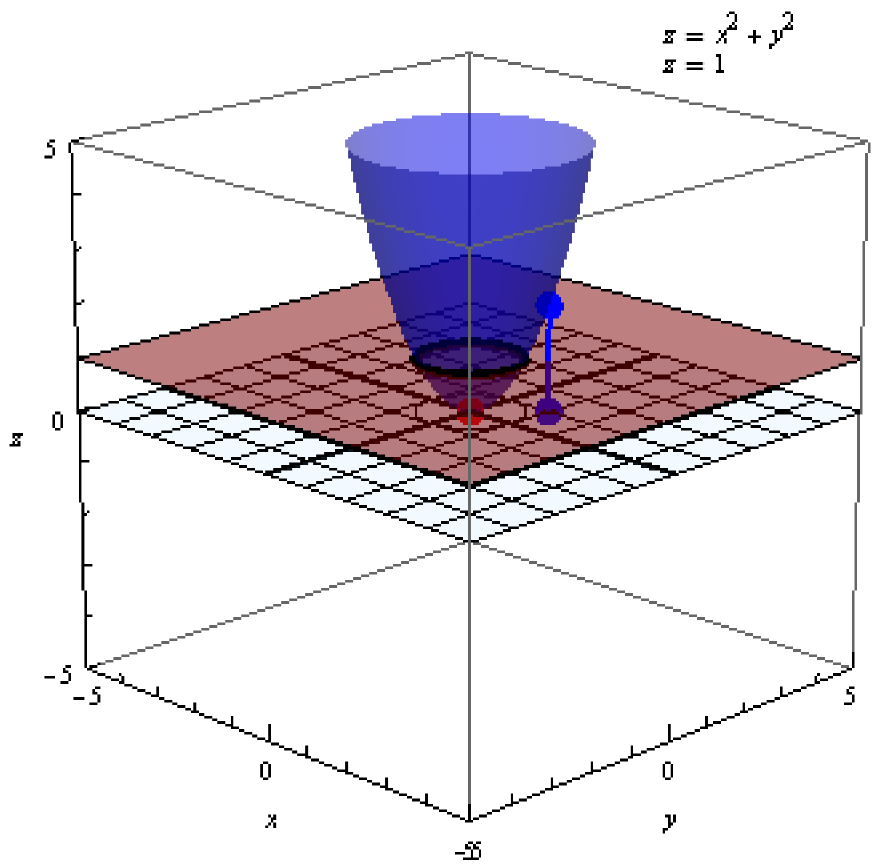




Then, we look to the intersection of these two equations:







Hence, $(0,0)$ produces $z = (0)^2 + (0)^2 = 0$ which is less than 1 and $(1,1)$ produces $z = (1)^2 + (1)^2 = 2$ which is greater than 1.

Therefore, we only need to create the following if statement:

```
if ( x * x + y * y <= 1 ) {
    // if true, the point is inside the circle
} else {
    // if false, the point is outside the circle
}
```

For $(0,0)$, we have the following conditional:

```
      x * x + y * y <= 1
(0) * (0) + (0) * (0) <= 1
         0 + 0 <= 1
           0 <= 1
            true
```

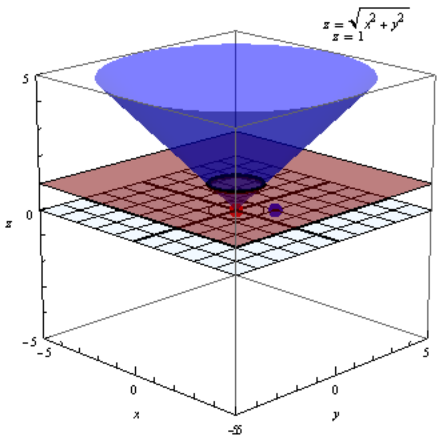
Hence, $(0,0)$ is inside the circle and the calculation above is exact.

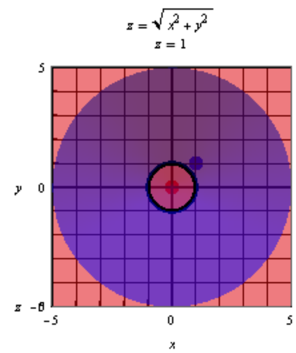
For $(1,1)$, we have the following conditional:

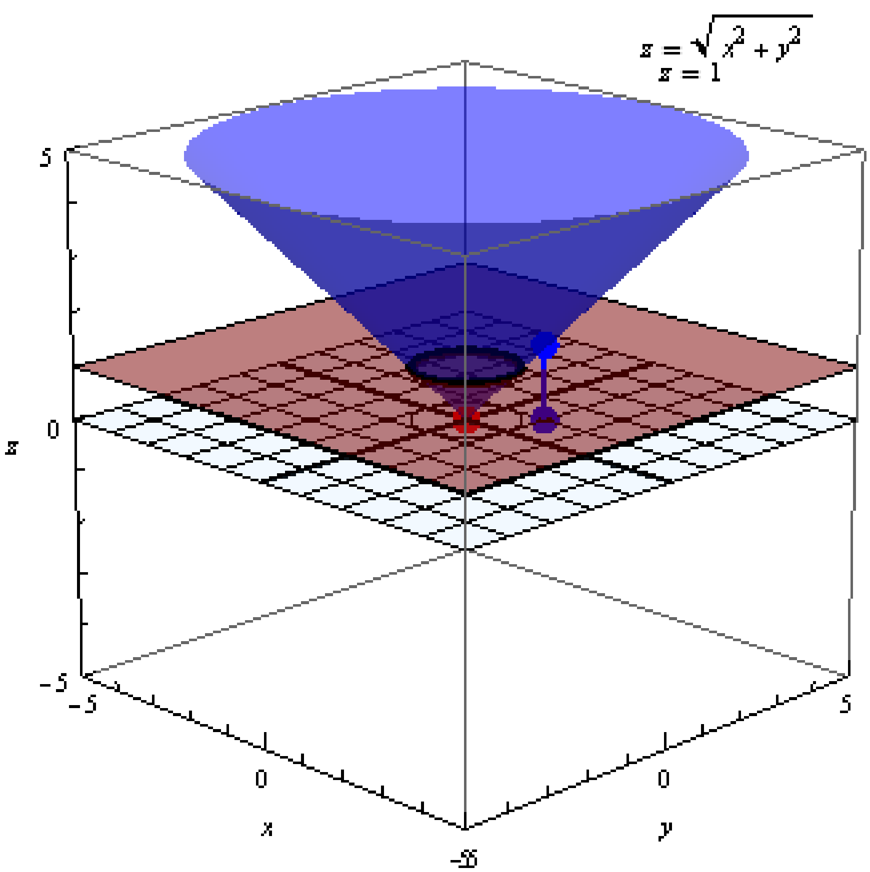
```
      x * x + y * y <= 1
(1) * (1) + (1) * (1) <= 1
         1 + 1 <= 1
           2 <= 1
           false
```

Hence, $(1,1)$ is outside the circle and the calculation above is exact.

For this problem, you could also be more complex and use the distance to each point from the origin such that $\text{distance}(x, y) = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$ and $\text{distance}(x, y) \leq 1 = \text{radius}$. Our graph in this case would be:







and our conditional would be:

```
if ( sqrt( x * x + y * y ) <= 1 ) {
    // if true, the point is inside the circle
} else {
    // if false, the point is outside the circle
}
```

For $(0,0)$, we have the following conditional:

```
      sqrt( x * x + y * y ) <= 1
  sqrt( (0) * (0) + (0) * (0) ) <= 1
          sqrt( 0 + 0 ) <= 1
            sqrt( 0 ) <= 1
                 true
```

Hence, $(0,0)$ is inside the circle and the calculation above is exact.

For $(1,1)$, we have the following conditional:

```
      sqrt( x * x + y * y ) <= 1
  sqrt( (1) * (1) + (1) * (1) ) <= 1
          sqrt( 1 + 1 ) <= 1
            sqrt( 2 ) <= 1
1.4142135623730950488016887242097 <= 1
                false
```

Hence, $(1,1)$ is outside the circle, but the calculation above is not exact. Therefore, we show the following exact calculation:

$$\sqrt{x^2 + y^2} \le 1$$

$$\sqrt{(1)^2 + (1)^2} \le 1$$

$$\sqrt{1+1} \le 1$$

$$\sqrt{2} \le 1$$

$$\text{false}$$

Hence, $(1,1)$ is outside the circle and the calculation above is exact.