COT 5507: Analytic Methods in Computer Science
Spring 2006

Assignment 1

Due: 16 Feb 2006, 5 pm.

1. (20 points) Prove that the sum of the cubes of the first n positive integers is $n^2(n+1)^2/4$, *using induction*.

2. (20 points) Solve the following recurrence *using the repertoire method*:

$$T(0) = 1$$
$$T(n) = T(n-1) + n + 2^n, \quad n > 0$$

3. (20 points) Exercise 2.20.

4. (20 points) In a variant of the Josephus problem, let every third person be eliminated, starting from person 3. Write a recurrence to determine the position of the survivor. You *need not* determine a closed form expression for it.

5. (20 points) Solve the following recurrence using *summation factors*:

$$T(0) = 1$$
$$3(n+1)^2\,T(n) = n^2\,T(n-1) + n\,(2/3)^n, \quad n > 0$$